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Analysis of the Portuguese Regulatory Model for Energy Distribution

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Executive Summary

The regulation of a company aims to achieve two main goals: efficiency maximization, by the regulated firm (or, in other words, cost minimization) and setting a price for the good/service produced that allows the regulated company to break even. This paper aims to analyze the current Portuguese regulatory model for the energy distribution sector, in order to realize if it meets the main objectives of regulation.

In Section 1, I analyze the company's incentives to maximize efficiency. I want to know if a company, acting in order to maximize its profits, is interested in minimizing its costs. This analysis seems very relevant, as one of the main purposes of a regulatory model is to link the company's profits with its costs in such a way that profit maximization implies cost minimization. Subsequently, I analyze the regulated company's profit. As one of the main objectives of a regulatory model is to allow zero profit to the regulated firm, it seems important to comprehend whether the present Portuguese regulatory model permits the regulated company to break even. I conclude that the current Portuguese regulatory model for energy distribution not only doesn't incentivize the regulated company to be efficient but also doesn't allow the regulated company to break even. Hence, I propose a new regulatory model that meets these main objectives. However, this proposed model can't take place yet, because there is not enough data. So, I also propose a new model that should be implemented until there is enough data for the first proposed model to take place.

Section 2 analyzes efficiency maximization in a more specific context. I intend to know whether the regulated company is incentivized to implement a new production process,

that allows the company to produce with lower costs, when the production process is available or if there is some incentive for the company to postpone its implementation. I conclude that the company is often incentivized to delay the implementation of a new production process and, therefore, I propose a model that incentivizes the company to apply a new production process whenever it is available.

Brief Description of the Energy Distribution Sector and of the Current Portuguese Regulatory Model

Specificities of the Sector

In Portugal, energy is distributed through EDP Distribuição. This company operates in all the Portuguese territory. The nature of the service led the company to divide the country into 14 geographical areas. To each of these geographical areas corresponds a network responsible for the electric power distribution. The technology used by the company to distribute the energy is the same for all networks. An important issue in this sector is the lack of control, by the distributing company, on the quantities to produce. In fact, the regulator imposes the company to distribute all the energy required by the market. Furthermore, given the nature of the service, the company cannot distribute more energy than what is required by the market, nor can the company create stocks. Thus, the company is constrained to distribute exactly the amount of energy requested by the market.

Brief Description of the Current Portuguese Regulatory Model

Presently, the Portuguese regulatory model is the Price-Caps, which consists in fixing a maximum price for the service. Indeed, price cap regulation adjusts the operator's prices according to the price cap index that reflects the overall rate of inflation in the economy and the ability of the operator to gain efficiencies. For this reason, price-cap regulation is sometimes called “CPI-X” after the basic formula employed to set price caps. This takes the rate of inflation, measured by the Consumer Price Index, and subtracts expected efficiency savings – X. This savings may occur due to the development of new technologies or to the increase of the firm’s efficiency. The idea behind this regulatory model is that, as prices are fixed, the only way that the company can increase its profits is by minimizing its costs (or, in other words, maximizing its efficiency). Furthermore, the maximum price set by the regulator should be the minimum unitary cost that the firm can attain. So, this model seems to accomplish the main goals of the regulation of a company. However, one should note that the X-factor (the expected efficiency savings) plays a key role in the model, so the way that this parameter is calculated may influence the conduct of the company. In fact, depending on how the X-Factor is calculated, it may be the case that the company has no longer incentives to minimize its costs.

Calculation of the X-Factor in Portugal

In order to set the maximum price allowed in a given period, the regulator (ERSE - Entidade Reguladora dos Serviços Energéticos) examines the efficiency of all the 14 networks in all previous periods.

Generally speaking, efficiency is defined as the ratio between the quantity produced and the cost of production. However, when calculating the efficiency of the networks, the regulator takes into account some exogenous factors, that is, factors that influence costs, but on which the company has no control. As this market is characterized by economies of scale, and since the quantity to produce in each network is exogenous, the regulator cannot compare, directly, the ratio (volume/production cost) between networks. In fact, it is expected that networks that produce larger amounts would have lower unit costs, ceteris paribus. But scale economies are not the only exogenous factor that should be taken into account. Thus, the regulator estimates, through an econometric model, the cost function of the network, which will depend not only on the quantity produced, but also on the other exogenous variables that the regulator considers to be relevant. Afterwards, the regulator uses the estimated cost function to obtain, for each network, the expected production cost. The network with the higher (expected cost/observed cost) ratio is considered to be the most efficient network. In subsequent regulatory periods, the regulator will demand all networks to have an (expected cost/observed cost) ratio at least as high as the most efficient network's ratio. A pertinent question arises: is the estimated cost function sufficient to explain the minimum cost of production, for all the networks? This will only happen if the regulator uses a large number of regressors in the econometric model, since there are many exogenous factors affecting the cost, namely the dimension of the geographical area, the client concentration, the consumption per household, the consumption of Low Voltage (LV) energy per household, the proportion of energy distributed in LV, the proportion of the LV Network, the proportion of the underground cables, the utilization of the installed power, the proportion of customers in rural areas, the quantity of non distributed energy, the price of the production factors and even the soil characteristics. Currently, the Portuguese regulatory model only considers, as exogenous factors, the extent of the network, the proportion of the LV network and the proportion of energy distributed in LV. Several exogenous factors are left because the stochastic specification of the econometric model is not appropriate to include them. Since there are many exogenous factors not considered in the cost function, it seem unreasonable to compare the expected cost (obtained with the estimated cost function) with the observed cost. In fact, those exogenous factors, disregarded in the estimation of the cost function, may be the cause of some differences between the predicted and the observed cost. If, for instance, production factors' prices are higher in a given geographical region, it is expected that the costs of the network serving that region will be higher than those for a network serving another region, ceteris paribus. Yet, as the regulator does not consider the price of the production factors as an exogenous factor in the estimation of the cost function, the regulator will consider the network that operates in the geographical region that has more expensive production factors to be inefficient.

The regulator assumes that the potential efficiency of each network does not decrease with time, i.e., the regulator considers that if a certain network has reached a certain level of efficiency in a given period, the same network can achieve at least the same level of efficiency, in any subsequent period. Another assumption adopted by the regulator is that all networks have equal efficiency potential, after some of the exogenous factors mentioned above are removed. Thus, the regulator assumes that if a certain network has reached a certain level of efficiency in a given period then, in that period, any network can achieve, at least, that level of efficiency. Combining these two assumptions, the regulator assumes that if a certain network has reached a certain level of efficiency in a given period, any network can achieve that level of efficiency in that period, or in any subsequent period. Thus, the method used by the regulator to set the maximum allowed price is to require, in every period, all networks to have, at least, the maximum level of efficiency verified in the past for any of the networks.

In general, with the aim of providing zero profit to the company, the regulator sets the revenue in the same amount of costs that the company would face if all networks produced with the maximum level of efficiency verified in the past by any of the networks. If the company presents a lower cost, it will profit the difference of the costs. The company will face a loss if the production cost is higher than the fixed revenue. Of course, if the company's production cost is equal to the fixed revenue, it will break even.

SECTION 1 – ANALYSIS OF THE COMPANY'S EFFICIENCY

1.1 Brief Critic of the Current Regulatory Model

As stated above, this regulatory model results from two assumptions made by the regulator: i) the potential efficiency of each network does not decrease with time; ii) all networks have the same potential efficiency.

In general, technological progress has allowed companies to become more efficient with time. Thus, the first assumption seems quite reasonable.

But, if the first assumption is consistent with what is observed in reality, the same does not apply for the second assumption. In fact, although the production function is identical for all the networks, several factors may allow networks different potential efficiency. Indeed, the prices of some production factors may vary according to the geographical region. Outsourcing usually have different prices depending on the geographical area of the country. Furthermore, even the soil characteristics affect the level of costs necessary to distribute energy. It is, actually, cheaper to install an electrical cable in a plain than in a mountainous region. Since the topography of the 14 geographical areas where the networks operate are different, it is natural that potential efficiency is not the same for all networks. Thus, the assumption adopted by the regulator does not seem to fit with reality. As the current regulatory model was developed according to this assumption, it is possible that the incentives for the company to maximize its efficiency in all networks do not occur in a context where this assumption does not hold. Therefore, I will analyze the behavior of a company that faces the current regulatory model, but whose networks have different potential efficiency.

1.2 Analysis of the Company's Behavior

I will consider a company with n networks (network 1, network 2, …, network n). Network 1 has greater potential efficiency than network 2, which has greater potential efficiency than network 3, and so on. I will consider that the company uses h production factors $(X_1, X_2, \dots, X_h)$ to distribute an exogenously set quantity of energy (Q). For simplicity, I will consider that the amount of energy distributed in each network is constant over time. This hypothesis is not very restrictive, given the rigidity that characterizes the demand curve in this market. I will also consider that the price of the production factors is constant over time (although it may vary within networks).

Let:

P^i be the allowed price set by the regulator, in period i

Q_j be the quantity produced in network j

$X^i_{f,j}$ be the amount of the production factor X_f used in network j, in period i

$W_{f,j}$ be the price of the production factor X_f for network j

The production function $\left(F(X_1, X_2, \dots, X_h)\right)$ is, by assumption, the same for all networks.

The company's profit is, in each period,

$$\pi^{\mathrm{i}} = \mathrm{P}^{\mathrm{i}}\mathrm{Q} - \sum_{j=1}^{n}\sum_{b=1}^{h} \mathrm{X}^{\mathrm{i}}_{\mathrm{b,j}}\mathrm{W}_{\mathrm{b,j}}$$

The firm will maximize profit, subject to the constraint that it has to produce, in each network, a given amount, i.e.,

$$F\left(X_{1,j}, X_{2,j}, \dots, X_{h,j}\right) = Q_j, \quad \forall\, j = 1,2, \dots, n$$

Company's revenue is set by the regulator at the beginning of the regulatory period. As stated above, the regulator requires all networks to have, at least, the same efficiency as the most efficient network in the past. The level of efficiency is defined by the regulator as the ratio of the estimated cost (obtained with the estimated cost function) and the observed cost.

Let

E_c^i be the level of efficiency verified in network c, in period i

C_c^i be the observed cost in network c, in period i

F_c be the estimated cost in network c

$\bar{C}_c^i$ be the minimum production cost for network c, in period i

The regulator defines the level of efficiency as follows:

$$E_c^i = \frac{F_c}{C_c^i}$$

It is noteworthy that the term "potential efficiency" refers to the possibility of the network to increase the relative difference between the estimated cost and the observed cost. Therefore, the term "potential efficiency" is not just about the company's ability to produce with low level of costs in a given network. Indeed, a network can produce at higher costs than other and still be more efficient if the cost estimated for the first network is greater than that estimated for the latter.

The regulator requires all networks to have at least the same efficiency as the most efficient network in the past. Thus, the regulator states that if a certain network has the same efficiency of the most efficient network in the past, it will break even. If the efficiency of the network exceeds the maximum efficiency occurred in the past, the network will profit. Similarly, if the network efficiency is less than the maximum efficiency occurred in the past, the network will face a loss.

Note: The estimated cost of the networks depends only on the quantity and on some exogenous factors that are, by assumption, constant for each network, namely the network extension. Thus, the estimated cost for each of the networks will be constant over time, which, at first glance, does not seem to fit in a model of price-cap regulation. However, if the firm is operating, in a network, with a lower cost than the estimated cost, it's efficiency will be greater than 100%. This implies that, in subsequent periods, the allowed revenue is also lower than the estimated cost. Therefore, the estimated cost is merely a reference to the extent that the level of efficiency imposed by the regulator could result in lower (or greater) revenue than the estimated cost.

So, let

$\dot{E}^i$ be the maximum level of efficiency verified in any network, till period i

R_c^i be the allowed revenue for network c, in period i

The regulator imposes that, for all networks, revenues are equal to the cost that the network would face if operating with the same level of efficiency as the maximum efficiency obtained, in the past, by any network. If a network operates with the same efficiency as the maximum efficiency, verified in the past, by any network, it will face the following cost:

$$\frac{F_c}{C_c^i} = \dot{E}^{i-1} <=> C_c^i = \frac{F_c}{\dot{E}^{i-1}}$$

Hence, in order to allow zero profit to the network operating with the maximum level of efficiency verified in the past, the regulator sets the network's revenue as the cost that the network faces when producing at that level of efficiency, i.e.:

$$R_c^i = \frac{F_c}{\dot{E}^{i-1}}$$

The profit function, defined above, can be written as follows:

$$\pi^{\mathrm{i}} = \sum_{c=1}^{n} R_c^i - \sum_{c=1}^{n}\sum_{b=1}^{h} \mathrm{X}_{\mathrm{b,c}}^{\mathrm{i}} \mathrm{W}_{\mathrm{b,c}}$$

Replacing

$$C_c^i = \sum_{b=1}^{h} \mathrm{X}_{\mathrm{b,c}}^{\mathrm{i}} \mathrm{W}_{\mathrm{b,c}}$$

$$R_c^i = \frac{F_c}{\dot{E}^{i-1}}$$

One obtains:

$$\pi^{\mathrm{i}} = \sum_{c=1}^{n} \left(\frac{F_c}{\dot{E}^{i-1}} - C_c^i \right)$$

The company will maximize profit, subject to the constraint of producing, in each network, an exogenously set amount, i.e.:

$$Max \qquad \pi^{\mathrm{i}} = \sum_{j=1}^{n} \left(\frac{F_c}{\dot{E}^{i-1}} - C_c^i \right)$$

s.t.

$$F(X_{1,j}, X_{2,j}, \dots, X_{h,j}) = Q_j, \quad \forall\, j = 1,2, \dots, n$$

The company's constraints may be replaced by:

$$C_j^i \geq \bar{C}_j^i, \quad \forall\, j = 1,2, \dots, n$$

I will start by proving that $\boldsymbol{F(X_{1,j}, X_{2,j}, \dots, X_{h,j}) = Q_j \Rightarrow C_j^i \geq \bar{C}_j^i}$

Let C_j^i be any cost that allows the production of the quantity Q_j,in network j,during period i.

By definition, since $\bar{C}_j^i$ is the minimum cost that allows the production of the quantity Q_j, in network j, during period i:

$$C_j^i \geq \bar{C}_j^i$$

Thus, one can conclude that $F(X_{1,j}, X_{2,j}, \dots, X_{h,j}) = Q_j \Rightarrow C_j^i \geq \bar{C}_j^i$

I will, now, examine whether $\boldsymbol{C_j^i \geq \bar{C}_j^i \Rightarrow F(X_{1,j}, X_{2,j}, \dots, X_{h,j}) = Q_j}$

The expression above is not true, i.e., the fact that a production cost in network j, during period i is greater or equal than the minimum cost to produce the quantity Q_j in the same network, during the same period does not imply that the quantity produced is Q_j. However, assuming that any level of costs above the minimum needed to produce a given quantity can be used to produce that quantity, the company can choose any cost above the minimum needed to produce a given quantity Q_j, i.e., $C_j^i \geq \bar{C}_j^i$, and, afterwards, choose the production factors that verify:

$$F(X_{1,j}, X_{2,j}, \dots, X_{h,j}) = Q_j$$

$$C_j^i = \sum_{\mathrm{b}=1}^{\mathrm{h}} \mathrm{X}_{\mathrm{b,j}}^{\mathrm{i}} \mathrm{W}_{\mathrm{b,j}}$$

Hence, as stated, the company's constraints

$$F(X_{1,j}, X_{2,j}, \dots, X_{h,j}) = Q_j, \quad j = 1,2, \dots, n$$

May be replaced by

$$C_j^i \geq \bar{C}_j^i, \qquad j = 1,2, \dots, n$$

So, the problem of a company that maximizes profit, in each period, is:

$$\text{Max} \quad \pi^{\mathrm{i}} = \sum_{c=1}^{n} \left(\frac{F_c}{\dot{E}^{i-1}} - C_c^i \right)$$

s.t.

$$C_c^i \geq \bar{C}_c^i, \qquad \forall\, c = 1,2, \dots, n$$

The Lagrangean is:

$$\mathcal{L} = \sum_{c=1}^{n} \left(\frac{F_c}{\dot{E}^{i-1}} - C_c^i \right) + \sum_{c=1}^{n} \lambda_c \left(C_c^i - \bar{C}_c^i \right)$$

The Kuhn-Tucker conditions are:

$$\frac{\partial \mathcal{L}}{\partial C_c^i} = 0, \qquad \forall\, c = 1,2, \dots, n$$

$$\frac{\partial \mathcal{L}}{\partial \lambda_c} \geq 0, \qquad \forall\, c = 1,2, \dots, n$$

$$\lambda_c \geq 0, \qquad \forall\, c = 1,2, \dots, n$$

$$\lambda_c \frac{\partial \mathcal{L}}{\partial \lambda_c} = 0, \qquad \forall\, c = 1,2, \dots, n$$

Solving the system, one obtains:

$$\frac{\partial \mathcal{L}}{\partial C_c^i} = 0 <=> \lambda_c = 1$$

$$\lambda_c \frac{\partial \mathcal{L}}{\partial \lambda_c} = 0 <=> C_c^i = \bar{C}_c^i, \qquad \forall\, c = 1,2, \dots, n$$

Thus, one concludes that a company whose objective is to maximize profit in each period will minimize the production costs on all the networks.

However, the objective of a company is not to maximize its profit in each period, but rather to maximize the expected present value of the profit. The present value of the company's profit is:

$$\pi = \sum_{\mathrm{i}=1}^{\infty} \delta^{\mathrm{i}} \left[\sum_{j=1}^{n} \left(\frac{F_c}{E_1^{i-1}} - C_c^i \right) \right]$$

Where

$$\delta = \frac{1}{(1+r)^t} \ , \quad where$$

r is the annual interest rate

t is the number of years in the regulatory period.

Note: As mentioned before, the regulator requires all networks to be, at least, as efficient as the most efficient network in the past (not necessarily in the previous year). By setting, in the profit function, the maximum efficiency in the past as the efficiency obtained by the network with the greatest potential for efficiency in the previous period, I am assuming not only that the network with the greatest potential for efficiency is, indeed, the most efficient network, but also that the level of efficiency in this network does not decrease with time. I may assume that the efficiency level in the network with the greatest potential for efficiency does not diminish over time, without loss of generality, in the sense that the company has no incentive to reduce the efficiency of that network. In fact, if the company increases the production cost in that network, it will face a loss in the amount of the difference between the current cost and the previous cost. This loss is the only impact of increasing the production cost in the most efficient network, as it will not affect the revenue of the subsequent periods, defined as the ratio between the estimated costs and the maximum level of efficiency attained by any network in the past.

Replacing, in the profit function:

$$E_1^{i-1} = \frac{F_1}{C_1^{i-1}}$$

The problem of the company can be written as follows:

$$Max \qquad \pi = \sum_{\mathrm{i}=1}^{\infty} \delta^{\mathrm{i}} \left[\sum_{c=1}^{n} \left(C_1^{i-1} \frac{F_c}{F_1} - C_c^i \right) \right]$$

s.t.

$$C_c^i \geq \bar{C}_c^i, \qquad \forall\, c = 1,2, \dots, n$$

The Lagrangean is:

$$\mathcal{L} = \sum_{\mathrm{i}=1}^{\infty} \left\{ \delta^{\mathrm{i}} \left[\sum_{c=1}^{n} \left(C_1^{i-1} \frac{F_c}{F_1} - C_c^i \right) \right] + \sum_{c=1}^{n} \lambda_{ci} \left(C_c^i - \bar{C}_c^i \right) \right\}$$

The Kuhn-Tucker conditions are:

$$\frac{\partial \mathcal{L}}{\partial C_1^i} = 0$$

$$\frac{\partial \mathcal{L}}{\partial C_c^i} = 0, \qquad \forall\, c = 2, \dots, n$$

$$\frac{\partial \mathcal{L}}{\partial \lambda_{ci}} \geq 0, \qquad \forall\, c = 1,2, \dots, n$$

$$\lambda_{ci} \geq 0, \qquad \forall\, c = 1,2, \dots, n$$

$$\lambda_{ci} \frac{\partial \mathcal{L}}{\partial \lambda_{ci}} = 0, \qquad \forall\, c = 1,2, \dots, n$$

Solving the system, one obtains:

$$\frac{\partial \mathcal{L}}{\partial C_1^i} = 0 <=> -\delta^i + \lambda_{1i} + \delta^{i+1}\left(\sum_{a=1}^{n}\frac{F_a}{F_1}\right) <=> \lambda_{1i} = \delta^i - \delta^{i+1}\left(\sum_{a=1}^{n}\frac{F_a}{F_1}\right)$$

Notice that

$$\lambda_{1i} = \delta^i - \delta^{i+1}\left(\sum_{a=1}^{n}\frac{F_a}{F_1}\right)$$

May violate one of the system's conditions:

$$\lambda_{1i} \geq 0$$

This happens because the Lagrangean is defined assuming that, as the company has no incentive to increase the production cost in the most efficient network, revenue will be set by the regulator, in each period, as the ratio between the estimated costs and the efficiency level achieved by that network in the previous period. Logically, however, if the company reduces the level of efficiency in the most efficient network in a given period, the revenue for the following regulatory period will not be a function of this new (and lower) level of efficiency, but rather a function of the maximum level of efficiency that any network have reached in the past. The Lagrangean above does not take this fact into account, that is, it assumes that if the company reduced the efficiency level of the most efficient network, the revenue for the following period would increase. Thus, to be precise, the Lagrangean is:

$$\mathcal{L} = \sum_{\mathrm{i}=1}^{\infty}\left\{\delta^{\mathrm{i}}\left[\sum_{c=1}^{n}\left(F_c * min\left\{\frac{C_1^{i-1}}{F_1};\frac{C_2^{i-1}}{F_2};\ldots;\frac{C_n^{i-1}}{F_n};\frac{C_1^{i-2}}{F_1};\ldots;\frac{C_n^{i-2}}{F_n};\frac{C_1^{0}}{F_1};\ldots;\frac{C_n^{0}}{F_n}\right\} - C_c^i\right)\right] + \sum_{c=1}^{n}\lambda_{ci}\left(C_c^i - \bar{C}_c^i\right)\right\}$$

Given the evident difficulty of studying this Lagrangean, I will address the problem differently.

Let $\overline{\mathrm{E}}_{\mathrm{c}}^{\mathrm{i}}$ be the maximum efficiency level that network c can attain in period i, that is:

$$\bar{E}_c^i = \frac{F_c}{\bar{C}_c^i}$$

The following analysis is based on the two following assumptions:

Assumption 1: Network 1 is the network with the highest potential for efficiency. Network 2 is the 2nd network with the highest potential for efficiency. Network n is the nth network with the highest potential for efficiency. And so on, i.e.:

$$\bar{E}_c^i > \bar{E}_{c+1}^i, \qquad \forall\, c \in \mathbb{N}:\ c < n$$

Assumption 2: In the initial period, network 1 is, indeed, the most efficient network, that is:

$$E_1^0 > E_c^0, \qquad \forall\, c \in \mathbb{N}:\ 1 < c \leq n$$

I will begin by distinguishing two mutually exclusive scenarios that cover the whole problem.

Scenario 1: In the initial period, the level of efficiency attained by network 1 is greater than the maximum level of efficiency attainable by any other network in the following period. i.e.:

$$E_1^0 > \bar{E}_c^1, \qquad \forall\, c \in \mathbb{N}:\ 1 < c \leq n$$

Scenario 2: There is, at least, one network that can attain, in period 1, the efficiency level attained by network 1 in period 0, that is:

$$E_1^0 \leq \bar{E}_2^1$$

I will analyze the behavior of the company in both scenarios.

Scenario 1: In the initial period, the level of efficiency attained by network 1 is greater than the maximum level of efficiency attainable by any other network in the following period

It's straightforward to see that, in this scenario, the firm is incentivized to maximize efficiency in networks $2, 3, \dots, n$. Indeed, by increasing the efficiency level in these networks, the company will increase its profits (or, in other words, minimize its losses). This increase of profits is, actually, the only impact from increasing the efficiency level in those networks, since it doesn't influence the revenue, in the extent that this new efficiency level is, still, lower than the one attained by network 1 in the previous period.

But if the firm is incentivized to maximize efficiency in networks with low potential for efficiency, the same is not necessarily true when it comes to the network with the greatest potential for efficiency (network 1). If the company increases the efficiency level in network 1, it will profit, today, the variation of costs. In the following regulatory periods, however, the revenue allowed by the regulator will be set according to the new level of efficiency attained by network 1. The revenue allowed by the regulator is:

$$R^i = \sum_{c=1}^{n} \frac{F_c}{\dot{E}^{i-1}}$$

Replacing $\dot{E}^{i-1} = \frac{F_1}{C_1^{i-1}}$

$$R^i = \sum_{c=1}^{n} C_1^{i-1} \frac{F_c}{F_1} = C_1^{i-1} \frac{\sum_{c=1}^{n} F_c}{F_1}$$

Thus, a reduction in the production cost, in a given period, by network 1 (in the amount of ΔC_1) will cause a reduction in allowed revenues in the amount of $\Delta C_1 \frac{\sum_{c=1}^{n} F_c}{F_1}$, for the following periods. Hence, the present value of the revenue's decrease is:

$$\sum_{i=1}^{\infty} \delta^i \left(\Delta C_1 \frac{\sum_{c=1}^{n} F_c}{F_1} \right)$$

Notice that the expression above reflects the present value of the revenue's decrease, not the present value of the losses. Concerning networks $2,3,\ldots,n$ the revenue's decrease represents, indeed, a loss for the company, since the firm is virtually unable to reduce the production cost in those networks, in the extent that those networks are already operating with maximum efficiency. However, regarding network 1, the revenue's decrease has no impact in the company's profit. In fact, the firm can operate, in that network, with the same efficiency level, reducing the costs of network 1 in the same amount of the revenue reduction for that network. Hence, the present value of the loss is:

$$\sum_{i=1}^{\infty} \delta^i \left(\Delta C_1 \frac{\sum_{c=2}^{n} F_c}{F_1} \right)$$

As mentioned above, the present value of the gains is the variation of costs in network 1, that is, ΔC_1.

The company is incentivized to increase the level of efficiency in the network with the greatest potential for efficiency if (and only if) the present value of this conduct is not negative, i.e.:

$$\Delta C_1 \geq \sum_{i=1}^{\infty} \delta^i \left(\Delta C_1 \frac{\sum_{c=2}^{n} F_c}{F_1} \right) <=> \frac{1-\delta}{\delta} \geq \frac{\sum_{c=1}^{n} F_c - F_1}{F_1} <=>$$

$$<=> \frac{1-\delta}{\delta} \geq \frac{\sum_{c=1}^{n} F_c}{F_1} - 1 <=> \frac{F_1}{\sum_{c=1}^{n} F_c} \geq \delta$$

So, the company will only be willing to increase the efficiency level in network 1 if the estimated production cost for that network has a very significant weight in the total estimated production costs.

By definition:

$$\delta = \frac{1}{(1+r)^t}$$

Hence:

$$\frac{F_1}{\sum_{c=1}^{n} F_c} \geq \delta <=> \frac{F_1}{\sum_{c=1}^{n} F_c} \geq \frac{1}{(1+r)^t}$$

Solving for t:

$$t \geq \frac{\log\left(\frac{\sum_{c=1}^{n} F_c}{F_1}\right)}{\log(1+r)}$$

Since, for reasonable values of interest rate, $\log(1+r)$ will be very close to zero, and $\log\left(\frac{\sum_{c=1}^{n} F_c}{F_1}\right)$ will not be close to zero, unless the estimated cost for network 1 is close to the total estimated cost, $\frac{\log\left(\frac{\sum_{c=1}^{n} F_c}{F_1}\right)}{\log(1+r)}$ will be very high.

For simplicity, I will consider that the number of years in a regulatory period (t) is an integer. This assumption not only simplifies the analysis, but also facilitates the application of the regulatory model. The following table shows, for a given number of years in a regulatory period, the minimum ratio (estimated costs in network 1/total estimated costs) that satisfies the inequality, resulting in incentives for the company to increase efficiency in the network with the greatest potential for efficiency. In this table I have considered an annual interest rate of 15%, which is considerably higher than the current interest rates in the market.

T	$\frac{F_1}{\sum_{c=1}^{n} F_c}$ (%)
3	66
4	58
5	50
10	25
25	10

Thus, we find out that, for the length of the current Portuguese regulatory period (3 years), the inequality will only hold if the ratio (estimated costs in network 1/total estimated costs) is greater or equal than 66%, which, in practical terms, given that there are several networks, each one serving a defined geographical region, is inconceivable. Notice that this value (66%) was obtained considering a significantly high interest rate. The use of a lower interest rate would require a higher ratio (estimated costs in network 1/total estimated costs) for the inequality to hold. Furthermore, even for a relatively long regulatory period (10 years, more than three times the number of years in the current Portuguese regulatory period), the minimum ratio (estimated costs in network 1/total estimated costs) necessary to hold the inequality is still very high (25%). Only too extensive regulatory periods could create incentives for the company to increase its efficiency in the network with the greater potential efficiency, supported by a reasonable ratio (estimated costs in network 1/total estimated costs).

Therefore, one concludes that one possible way to incentivize the company to increase the efficiency of its network with the greatest potential for efficiency is to set quite extensive regulatory periods. Nevertheless, this is not realistic as it causes many other problems. In fact, the prediction of demand is the more difficult the more extensive the regulatory period is. Moreover, the estimation of technological progress is rather difficult to perform for long regulatory periods.

One can conclude that, in this scenario, where the level of efficiency attained by network 1, in the initial period, is greater than the maximum level of efficiency attainable by any other network in the following period, the company will maintain the production cost in the network with the greatest potential for efficiency and minimize the production cost in the remaining networks. Hence, this regulatory model allows, in this scenario, productive inefficiencies.

It is also worth mentioning that, in this scenario, the company faces losses.

Scenario 2: There is, at least, one network that can attain, in period 1, the efficiency level attained by network 1 in period 0

It is, in this scenario, straightforward to see that the firm has an incentive to increase the level of efficiency in all the networks to, at least, the efficiency level verified in the network with the greatest potential for efficiency, i.e., network 1. In fact, in the period where the reduction of the production cost occurs, the company increases its profits. In addition to this increase in profits, reducing the production cost in the networks will only have an impact in the allowed revenue if the efficiency of one of the networks increases to a higher level than the one verified in network 1. Hence, it is assured that the company will increase efficiency level in networks $2, 3, \dots, n$, if that is possible. The question is: what will the increment be? Nevertheless, as long as the efficiency level in a network is lower than the one verified in network 1, the firm will increase the efficiency level in that network. Hence, two possibilities arise: the company can increase the efficiency level in networks $2, 3, \dots, n$ to the same level of efficiency verified in network 1 or to a higher level. In any case, the company has, again, two options: maintain the efficiency level in network 1 or increase this level of efficiency. I will now analyze, from the perspective of the company's profit, the 4 possible cases, in this scenario.

Case 1: The company increases the efficiency level in networks $2, 3, \dots, n$ to the same level of efficiency verified in network 1

Thus, in this case,

$$E_j^1 = E_1^0, \quad \forall\, j = 2,3, \dots, n: \;\; \bar{E}_j^1 \geq E_1^0$$

$$E_j^1 = \bar{E}_j^1, \quad \forall\, j = 2,3, \dots, n: \;\; \bar{E}_j^1 < E_1^0$$

Case 1.1: The company maintains the efficiency level in network 1

In this case, the company's profit is:

$$\pi^{\mathrm{i}} = \sum_{c=1}^{n} \left(\frac{F_c}{\dot{E}^{i-1}} - C_c^i \right)$$

Let k be the maximum value that satisfies

$$\bar{E}_k^1 \geq E_1^0$$

$$\pi^{\mathrm{i}} = \sum_{c=1}^{k} \left(\frac{F_c}{\dot{E}^{i-1}} - C_c^i \right) + \sum_{c=k+1}^{n} \left(\frac{F_c}{\dot{E}^{i-1}} - C_c^i \right)$$

By definition, for the k networks operating with the same efficiency level as the maximum level verified in the past, $\frac{F_c}{\dot{E}^{i-1}} = C_c^i$. Thus, the company's profit, in a given period i, will be:

$$\pi^{\mathrm{i}} = \sum_{c=k+1}^{n} \left(\frac{F_c}{\dot{E}^{i-1}} - C_c^i \right)$$

The present value of profits is:

$$\pi = \sum_{\mathrm{i}=1}^{\infty} \delta^{\mathrm{i}} \left[\sum_{c=k+1}^{n} \left(\frac{F_c}{\dot{E}^{i-1}} - C_c^i \right) \right]$$

As the maximum efficiency level verified in the past is always the same, $\dot{E}^{i-1}$ will be a constant, and it's value will be E_1^0. Moreover, since the company maximizes efficiency in the

$(n-k)$ networks with the lower potential for efficiency, $C_c^i = \frac{F_c}{\bar{E}_c^i}$. Hence, the company's profit can be written as follows:

$$\pi = \sum_{i=1}^{\infty} \delta^i \left[\sum_{c=k+1}^{n} \left(\frac{F_c}{E_1^0} - \frac{F_c}{\bar{E}_c^i} \right) \right]$$

Notice that, since the $(n-k)$ networks with lower potential for efficiency operate with a lower efficiency level than the maximum efficiency level verified in the past, they will face losses. Thus, the present value of profits is, in this case, negative.

Case 1.2: The company increases the efficiency level in network 1

This case can never be the one that provides the highest profit to the company. In fact, if the company increases the efficiency level in networks $2, 3, \dots, n$ to E_1^0 and, simultaneously, increase the efficiency level in network 1 to an efficiency level greater than E_1^0 (let's call it E_{New}), the firm may increase its profit by increasing the efficiency level in networks $2, 3, \dots, n$ to E_{New}, in the sense that, in doing so, the company increases the profit in the current regulatory period and does not cause any impact on the revenue allowed, by the regulator, for the following periods. Thus, this case is dominated by the case 2.2., discussed below. Indeed, in the case 2.2., the company increases the efficiency level in network 1, but also increases the efficiency level in the remaining networks to a higher level than the one verified by network 1 in period 0.

Case 2: The company increases the efficiency level in networks $2, 3, \dots, n$ to a higher level than the one verified in network 1, in period 0

Thus, in this case,

$$E_j^1 > E_1^0, \quad \forall\, j = 2,3, \dots, n: \;\; \bar{E}_j^1 > E_1^0$$

$$E_j^1 = \bar{E}_j^1, \quad \forall\, j = 2,3, \dots, n: \;\; \bar{E}_j^1 \leq E_1^0$$

Case 2.1: The company maintains the efficiency level in network 1

Just as the case 1.2., this case can never be the one that provides the highest profit to the company, as it is also dominated by the case 2.2. In fact, if the level of efficiency in a given network is greater than the efficiency level verified in the network with the greatest potential for efficiency, the company has an incentive to increase efficiency in the latter network to, at least, the highest level of efficiency observed in all other networks. Indeed, this increase in the efficiency level of network 1 will lead to an increase of profits, in the period at which the increase of the efficiency level occurs, and will not have any impact on future profits, as it does not affect the maximum allowed revenue, set by the regulator.

Case 2.2: The company increases the efficiency level in network 1

As explained above, if the efficiency level of a network is greater than the efficiency level in other network, the company should, if possible, increase the efficiency level of the less efficient network.

Hence, as long as $\exists\, j, m \in \{1,2,\dots,n\}$: $E_j^i > E_m^i$, the company should increase E_m^i to, at least, E_j^i, if that is possible, i.e., if $\bar{E}_m^i \geq E_j^i$. Furthermore, if the company operates in all networks with the same level of efficiency and if it is possible to increase the efficiency level in all the networks, the company should increase the efficiency level in all the networks to, at least, the minimum value of the maximum efficiency level attainable in all the networks, that is, if $E_j^i = E_c^i, \forall\, j, c \in \{1,2,3,\dots,n\}$ and, simultaneously, $E_c^i < \bar{E}_c^i, \forall\, c = 1,2,\dots,n$, the company should increase the efficiency level in all the networks to, at least, $min\{\bar{E}_1^i; \bar{E}_2^i; \dots; \bar{E}_n^i\}$ or, in other words, $\bar{E}_n^i$.

Therefore, one can conclude that the highest level of efficiency attainable in the network with less potential for efficiency (network n) is the minimum efficiency level that the company should choose in any of the networks, in that, if the firm chooses a lower level of efficiency, it will gain by raising the efficiency level.

Furthermore, in the case that $E_1^0 > \bar{E}_n^1$, the efficiency level attained by network 1 in the initial period becomes the lower limit of the efficiency level that the company should choose in the networks that can achieve that level of efficiency. In the networks that can't achieve the efficiency level E_1^0, the company should maximize efficiency.

Let, again, k be the maximum value that satisfies

$$\bar{E}_k^1 \geq E_1^0$$

The company will maximize efficiency in the $(n-k)$ networks with lower potential efficiency. Now, one only needs to know the efficiency level chosen for the remaining networks, being E_1^0 the minimum limit of the efficiency level in those networks, as shown above.

If it is good for the company to set the efficiency in the k networks with higher potential efficiency in a level greater than E_1^0, than it is good for the company to set the efficiency in the k networks with higher potential efficiency at $\bar{E}_k^1$.

I will start by analyzing whether the company prefers to set the efficiency level in the k networks with higher potential efficiency at E_1^0 or at $\bar{E}_k^1$. If the company prefers to set the efficiency level at E_1^0 in those networks, that will be the optimum efficiency level for the k networks with higher potential efficiency. If, on the contrary, setting the efficiency level at $\bar{E}_k^1$ is preferred to setting the efficiency level at E_1^0, then the optimum efficiency level for the k networks with higher potential efficiency will be greater or equal than $\bar{E}_k^1$.

If, in a given regulatory period, a company operates with efficiency level E_1^0 in the k networks with higher potential efficiency and increases the efficiency level, in those networks, to $\bar{E}_k^1$, it will gain, in that regulatory period, the difference between the costs it would face operating with efficiency level E_1^0 and the observed costs. In the following regulatory periods, the regulator will require all networks to have, at least, an efficiency level of $\bar{E}_k^1$. Thus, the company will only profit from this increase in efficiency in the regulatory period when the company increases the level of efficiency.

If the company operated with efficiency level E_1^0 in the k networks with higher potential efficiency, it would face a cost, in each of these networks, of:

$$\frac{F_c}{C_c^i} = E_1^0 <=> C_c^i = \frac{F_c}{E_1^0}$$

Operating with efficiency level $\bar{E}_k^1$ in the k networks with higher potential efficiency, the company faces a cost, in each of these networks, of:

$$\frac{F_c}{C_c^i} = \bar{E}_k^1 <=> C_c^i = \frac{F_c}{\bar{E}_k^1}$$

Thus, the gain of the company, due to the increase in the efficiency level of the k networks with higher potential efficiency, is:

$$\sum_{c=1}^{k}\left(\frac{F_c}{E_1^0} - \frac{F_c}{\bar{E}_k^1}\right) = \left(\frac{1}{E_1^0} - \frac{1}{\bar{E}_k^1}\right)\sum_{c=1}^{k} F_c$$

This new efficiency level will also have an impact on the future revenues allowed by the regulator. In fact, the allowed revenue, in each network, is:

$$R_c^i = \frac{F_c}{\dot{E}^{i-1}}$$

Being $\dot{\mathrm{E}}^{\mathrm{i}-1}$ The maximum level of efficiency attained by any network, till period $i - 1$.

The increase of $\dot{\mathrm{E}}$ will, hence, lead to a decrease in future revenues, for all networks.

The allowed revenue, in the period when the company increased the efficiency level in the k networks with higher potential efficiency, is:

$$R_c^{i-1} = \frac{F_c}{E_1^0}$$

In the following period, the allowed revenue will be:

$$R_c^i = \frac{F_c}{\bar{E}_k^1}$$

Thus, the present value of the decrease in the allowed revenues is:

$$\sum_{i=1}^{\infty} \delta^i \left[\sum_{c=1}^{n} \left(\frac{F_c}{E_1^0} - \frac{F_c}{\bar{E}_k^1} \right) \right]$$

Notice that the expression above reflects the present value of the revenue's decrease, not the present value of the losses. Concerning the $(n-k)$ networks with lower potential efficiency, the revenue's decrease represents, indeed, a loss for the company, since the firm is virtually unable to reduce the production cost in those networks, in the extent that those networks are operating with maximum efficiency. However, regarding the k networks with higher potential efficiency, the revenue's decrease has no impact in the company's profit. In fact, the firm can operate, in those networks, with the same efficiency level $(\bar{E}_k^1)$, reducing the costs of the k networks with higher potential efficiency in the same amount of the revenue reduction for those networks. Hence, the present value of the loss is:

$$\sum_{i=1}^{\infty} \delta^i \left[\sum_{c=k+1}^{n} \left(\frac{F_c}{E_1^0} - \frac{F_c}{\bar{E}_k^1} \right) \right] = \frac{\delta}{1-\delta} \left(\frac{1}{E_1^0} - \frac{1}{\bar{E}_k^1} \right) \sum_{c=k+1}^{n} F_c$$

The company is incentivized to increase the level of efficiency in the k networks with higher potential for efficiency if (and only if) the present value of the gains is greater (or equal) than the present value of losses resulted from this conduct, i.e.:

$$\left(\frac{1}{E_1^0} - \frac{1}{\bar{E}_k^1} \right) \sum_{c=1}^{k} F_c \geq \frac{\delta}{1-\delta} \left(\frac{1}{E_1^0} - \frac{1}{\bar{E}_k^1} \right) \sum_{c=k+1}^{n} F_c <=> \frac{1-\delta}{\delta} \geq \frac{\sum_{c=k+1}^{n} F_c}{\sum_{c=1}^{k} F_c} <=>$$

$$<=> \frac{1-\delta}{\delta} \geq \frac{\sum_{c=1}^{n} F_c - \sum_{c=1}^{k} F_c}{\sum_{c=1}^{k} F_c} <=> \frac{\sum_{c=1}^{k} F_c}{\sum_{c=1}^{n} F_c} \geq \delta$$

So, the company only has an incentive to reduce the production costs in the networks with higher potential efficiency if the proportion of the estimated costs of those networks in total estimated production costs is sufficiently large.

If the inequality above does not hold, the company will operate with efficiency level E_1^0 in all networks where that level is attainable, and will maximize efficiency in the remaining networks.

If, on the contrary, the inequality above holds, the company will maximize efficiency in networks $k, k+1, k+2, \ldots, n$, and will operate, in the remaining networks, with an efficiency level not lower than $\bar{E}_k^1$. It will be interesting to know whether the company has incentives to produce, in the $(k-1)$ networks with higher potential efficiency, with an efficiency level greater than $\bar{E}_k^1$.

If it's good for the company to set the efficiency level in the $(k-1)$ networks with higher potential efficiency in a level greater than $\bar{E}_k^1$, then it's good for the company to set the efficiency level, in those networks, at $\bar{E}_{k-1}^1$.

If, in a given regulatory period, the company increases efficiency level to $\bar{E}_{k-1}^1$ in the networks where that level is attainable, the firm will profit, in that regulatory period, the difference between the costs it would face operating with efficiency level $\bar{E}_k^1$ and the costs faced by operating with efficiency level $\bar{E}_{k-1}^1$, in the $(k-1)$ networks with higher potential efficiency. Thus, the gain of the company, due to the increase in the efficiency level of the $(k-1)$ networks with higher potential efficiency, is:

$$\sum_{c=1}^{k-1}\left(\frac{F_c}{\bar{E}_k^1}-\frac{F_c}{\bar{E}_{k-1}^1}\right)$$

Nevertheless, in following regulatory periods, the company will face a loss in the difference between the allowed revenues if the efficiency level did not increase and the allowed revenues after the efficiency increase, in networks $k, k+1, k+2, \dots, n$. It is noteworthy that the $(k-1)$ networks with higher potential efficiency will not suffer any loss in following periods, in the sense that, although allowed revenues decrease, production costs will decrease in the same amount. Hence, the present value of the loss is:

$$\sum_{i=1}^{\infty} \delta^i \left[\sum_{c=k}^{n} \left(\frac{F_c}{\bar{E}_k^1} - \frac{F_c}{\bar{E}_{k-1}^1} \right) \right]$$

The company is incentivized to increase the level of efficiency in the $(k-1)$ networks with higher potential for efficiency if (and only if) the present value of the gains is greater (or equal) than the present value of losses resulted from this conduct, i.e.:

$$\sum_{c=1}^{k-1} \left(\frac{F_c}{\bar{E}_k^1} - \frac{F_c}{\bar{E}_{k-1}^1} \right) \geq \sum_{i=1}^{\infty} \delta^i \left[\sum_{c=k}^{n} \left(\frac{F_c}{\bar{E}_k^1} - \frac{F_c}{\bar{E}_{k-1}^1} \right) \right] <=>$$

$$<=> \left(\frac{1}{\bar{E}_k^1} - \frac{1}{\bar{E}_{k-1}^1} \right) \sum_{c=1}^{k-1} F_c \geq \frac{\delta}{1-\delta} \left(\frac{1}{\bar{E}_k^1} - \frac{1}{\bar{E}_{k-1}^1} \right) \sum_{c=k}^{n} F_c <=> \frac{1-\delta}{\delta} \geq \frac{\sum_{c=k}^{n} F_c}{\sum_{c=1}^{k-1} F_c} <=>$$

$$<=> \frac{1-\delta}{\delta} \geq \frac{\sum_{c=1}^{n} F_c - \sum_{c=1}^{k-1} F_c}{\sum_{c=1}^{k-1} F_c} <=> \frac{\sum_{c=1}^{k-1} F_c}{\sum_{c=1}^{n} F_c} \geq \delta$$

If the previous inequality $\left(\frac{\sum_{c=1}^{k} C_c^f}{\sum_{c=1}^{n} C_c^f} \geq \delta \right)$ holds and the above inequality doesn't, the company will operate with efficiency level $\bar{E}_k^1$ in the networks where that level is attainable, and maximize efficiency in the remaining networks.

If both inequalities hold, the company will maximize efficiency in networks $k-1, k, k+1, k+2, \dots, n$, and operate, in the remaining networks, with an efficiency level not lower than $\bar{E}^1_{k-1}$. It will be interesting to know whether the company has incentives to produce, in the $(k-2)$ networks with higher potential efficiency, with an efficiency level greater than $\bar{E}^1_{k-1}$.

Generalizing, if it's good for the company to set the efficiency of the $(k-a)$ networks with higher potential efficiency in a level greater than $\bar{E}^1_{k-a}$, then it's good for the company to set the efficiency level, in those networks, at $\bar{E}^1_{k-a-1}$, $\forall\, a \in \mathrm{Z}$: $-1 \leq a \leq k-2$.

If, in a given regulatory period, the company increases the efficiency level to $\bar{E}^1_{k-a-1}$, in the networks where that efficiency level is attainable, it will profit, in that period, the difference between the costs it would face operating with efficiency level $\bar{E}^1_{k-a}$ and the costs obtained by producing with efficiency level $\bar{E}^1_{k-a-1}$, in the $(k-a-1)$ networks with higher potential efficiency. Thus, the gain of the company, due to the increase in the efficiency level of the $(k-a-1)$ networks with higher potential efficiency, is:

$$\sum_{c=1}^{k-a-1} \left(\frac{F_c}{\bar{E}^1_{k-a}} - \frac{F_c}{\bar{E}^1_{k-a-1}} \right)$$

Nevertheless, in following regulatory periods, the company will face a loss in the difference between the allowed revenues if the efficiency level did not increase and the allowed revenues after the efficiency increase, in networks $k-a, k-a+1, k-a+2, \dots, n$. It is noteworthy that the $(k-a-1)$ networks with higher potential efficiency will not suffer any loss in following periods, in the sense that, although allowed revenues decrease, production costs will decrease in the same amount. Hence, the present value of the loss is:

$$\sum_{i=1}^{\infty} \delta^i \left[\sum_{c=k-a}^{n} \left(\frac{F_c}{\bar{E}_{k-a}^1} - \frac{F_c}{\bar{E}_{k-a-1}^1} \right) \right]$$

The company is incentivized to increase the level of efficiency in the $(k-a-1)$ networks with higher potential for efficiency if (and only if) the present value of the gains is greater (or equal) than the present value of losses resulted from this conduct, i.e.:

$$\sum_{c=1}^{k-a-1} \left(\frac{F_c}{\bar{E}_{k-a}^1} - \frac{F_c}{\bar{E}_{k-a-1}^1} \right) \geq \sum_{i=1}^{\infty} \delta^i \left[\sum_{c=k-a}^{n} \left(\frac{F_c}{\bar{E}_{k-a}^1} - \frac{F_c}{\bar{E}_{k-a-1}^1} \right) \right] <=>$$

$$<=> \left(\frac{1}{\bar{E}_{k-a}^1} - \frac{1}{\bar{E}_{k-a-1}^1} \right) \sum_{c=1}^{k-a-1} F_c \geq \frac{\delta}{1-\delta} \left(\frac{1}{\bar{E}_{k-a}^1} - \frac{1}{\bar{E}_{k-a-1}^1} \right) \sum_{c=k-a}^{n} F_c <=>$$

$$<=> \frac{1-\delta}{\delta} \geq \frac{\sum_{c=k-a}^{n} F_c}{\sum_{c=1}^{k-a-1} F_c} <=> \frac{1-\delta}{\delta} \geq \frac{\sum_{c=1}^{n} F_c - \sum_{c=1}^{k-a-1} F_c}{\sum_{c=1}^{k-a-1} F_c} <=> \frac{\sum_{c=1}^{k-a-1} F_c}{\sum_{c=1}^{n} F_c} \geq \delta$$

The company will operate with efficiency level $\bar{E}_{k-a-1}^1$ in the networks where that level is attainable, where a is the maximum integer that verifies the above inequality. In the remaining networks, the company will maximize efficiency. If the above inequality cannot be verified, for any a, the company will operate with efficiency level E_1^0 in the networks where that level is attainable and minimize costs in the remaining networks.

Thus, the company will only maximize efficiency in all networks if $\bar{E}_{k-a-1}^1 = \bar{E}_1^1$, that is, when $a = k-2$. That would imply that

$$\frac{F_1}{\sum_{c=1}^{n} F_c} \geq \delta$$

In other words, the company will only maximize the efficiency in all networks if the proportion of the estimated costs of the network with the greatest potential is quite significant, which, as shown previously, is inconceivable.

This conduct (case 2.2.) results in a profit, in the first period, of

$$\pi^0 = \sum_{c=1}^{k-a-1} \left(\frac{F_c}{E_1^0} - \frac{F_c}{\bar{E}_{k-a-1}^1} \right) + \sum_{c=k-a}^{n} \left(\frac{F_c}{E_1^0} - \frac{F_c}{\bar{E}_c^1} \right)$$

In following periods, the company's profit will be:

$$\pi^i = \sum_{c=k-a}^{n} \left(\frac{F_c}{\bar{E}_{k-a-1}^{i-1}} - \frac{F_c}{\bar{E}_c^i} \right)$$

Thus, the present value of the company's profit is:

$$\pi = \sum_{c=1}^{k-a-1} \left(\frac{F_c}{E_1^0} - \frac{F_c}{\bar{E}_{k-a-1}^1} \right) + \sum_{c=k-a}^{n} \left(\frac{F_c}{E_1^0} - \frac{F_c}{\bar{E}_c^1} \right) + \sum_{i=1}^{\infty} \delta^i \left[\sum_{c=k-a}^{n} \left(\frac{F_c}{\bar{E}_{k-a-1}^{i-1}} - \frac{F_c}{\bar{E}_c^i} \right) \right]$$

Previously I have shown that this case (2.2.) dominated both case 1.2. and case 2.1. The question is whether the case 2.2. is also better than the case 1.1.

It's easy to verify that the company's profit is greater in the case 2.2. In fact, in the case 1.1., the company operates with the maximum level of efficiency attained in the past, in the networks where this level is attainable, and maximizes efficiency in the remaining networks. In the case 2.2. the company will only operate with a greater efficiency level than the one verified in case 1.1. if that increases the company's profit. Thus, the case 2.2. allows, by construction, greater profits for the firm. Indeed, the behavior reflected in case 1.1. is a particular case of the behavior reflected in case 2.2.

Thus, in this scenario (scenario 2) the company's profit will be

$$\pi = \sum_{c=1}^{k-a-1} \left(\frac{F_c}{E_1^0} - \frac{F_c}{\bar{E}_{k-a-1}^1} \right) + \sum_{c=k-a}^{n} \left(\frac{F_c}{E_1^0} - \frac{F_c}{\bar{E}_c^1} \right) + \sum_{i=1}^{\infty} \delta^i \left[\sum_{c=k-a}^{n} \left(\frac{F_c}{\bar{E}_{k-a-1}^{i-1}} - \frac{F_c}{\bar{E}_c^i} \right) \right]$$

Since, for the networks with lower potential efficiency, $\bar{E}_c^i < \bar{E}_{k-a-1}^{i-1}$, the last component of profits will be negative. Thus, the present value of the profits will only be positive if there are no significant differences, in terms of potential efficiency, between the various networks or if the sum of the first two components of the present value of profits is large enough, which will only happen if the maximum efficiency level verified in the initial period is significantly lower than the efficiency attainable by the networks with high potential efficiency, in the following period. As the level of efficiency, in the initial period, was chosen by the company before knowing the rules of the game that it would face in the following period, this does not seem plausible. In fact, in the initial period, the company chose the efficiency level of the networks without knowing that, in the following period, this new regulatory model would take place. Hence, in principle, the regulated company faces losses, regardless of its behavior.

One can conclude that, in this scenario, the company will operate inefficiently in the networks with higher potential efficiency. However, this strategy adopted by the company does not prevent losses, only minimizes it. Hence, the current regulatory model implies, necessarily, losses for the regulated company. Moreover, the current regulatory model does not incentivize the company to maximize its efficiency in all networks. Indeed, by operating inefficiently in the networks with higher potential efficiency, the company can minimize the losses.

1.3. Conclusion

The current Portuguese regulatory model was created to incentivize the company to operate efficiently in all of its networks, assuming that the firm's networks have the same potential efficiency. However, this assumption does not seem to fit in reality. In fact, there are some exogenous factors that influence, differently, the networks' cost function. In order to realize if, in the failure of this hypothesis, the company was still incentivized to maximize its efficiency in all networks, I have analyzed the behavior of a firm whose networks have different potentials for efficiency. The study indicates that such company would operate inefficiently in the networks with higher potential for efficiency.

It follows that the (wrong) assumption in the basis of the current Portuguese regulatory model does not permit the model to achieve one of its main goals: to incentivize the regulated firm to operate efficiently, in all of its networks.

The fact that the company does not maximize efficiency in all of its networks implies that society, as a whole, supports a cost. The cost of society is the difference between the production cost observed in the networks where the company operates inefficiently and the minimum cost attainable by those networks, i.e.:

$$\sum_{c=1}^{k-a-2}\left(\frac{F_c}{\overline{E}_{k-a-1}^{i-1}}-\frac{F_c}{\overline{E}_c^{i}}\right)$$

This cost is supported by both the company and its consumers. The company's cost is the difference between the observed cost and the allowed revenue in the networks where the company operates efficiently, that is:

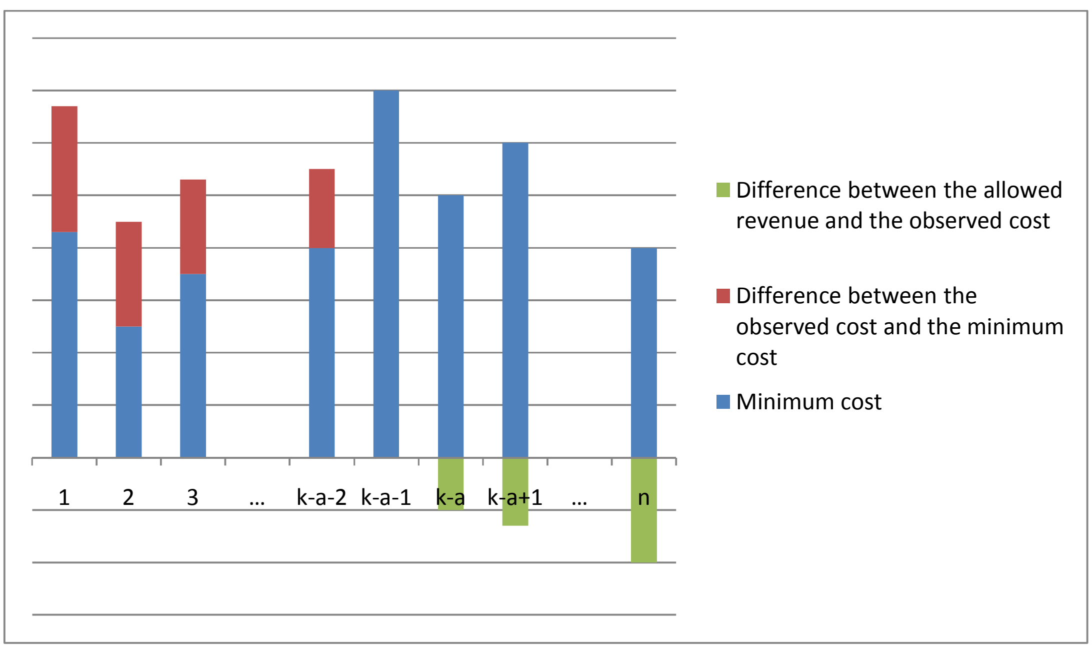
$$\sum_{c=k-a}^{n}\left(\frac{F_c}{\bar{E}_c^i}-\frac{F_c}{\bar{E}_{k-a-1}^{i-1}}\right)$$

The consumer's cost is the difference between the amount they pay and the amount that they would pay if the company operated efficiently in all its networks, or, in other words, the difference between the society's cost and the company's cost, i.e.:

$$\sum_{c=1}^{k-a-2}\left(\frac{F_c}{\bar{E}_{k-a-1}^{i-1}}-\frac{F_c}{\bar{E}_c^i}\right)-\sum_{c=k-a}^{n}\left(\frac{F_c}{\bar{E}_c^i}-\frac{F_c}{\bar{E}_{k-a-1}^{i-1}}\right)$$

Notice that the consumer's cost can be negative, that is, it is possible that consumers are benefiting from the inefficiencies caused by this regulatory model, in the sense that the amount paid by them may be less than the minimum cost required to distribute the requested energy. In that case, the company would bear not only the cost of society as a whole, but also subsidize lower prices for consumers.

The following chart illustrates the conduct of the company



It's easy to see that the company operates inefficiently in the $(k - a - 2)$ networks with higher potential efficiency. In those networks, the allowed revenue is equal to the production cost. In the remaining networks, the company operates efficiently. However, those are the networks that bring losses to the company. Indeed, in those networks, excluding network $(k - a - 1)$, the allowed revenue is less than the minimum attainable cost.

The cost supported by society is the sum of the red bars in the chart. The company's cost is the sum of the green bars and the consumer's cost is the difference between the red bars and the green bars.

1.4 Proposal for Action

Given that the current Portuguese regulatory model does not incentivize the company to operate efficiently in all of its networks, I will propose a new model to reach this goal.

The proposed model is quite similar to the current model, but without assuming that all networks have the same potential efficiency. Thus, the assumption of the proposed model is simply that the potential efficiency of each network does not decrease in time. In other words, the proposed model assumes that if a given network attains, in a given period, a given efficiency level, the same network can attain, in any subsequent period, at least the same efficiency level. Hence, the allowed revenue is, for each network, the ratio between the network's estimated cost and the maximum efficiency level attained by that network, in the past.

Thus, the company's profit, in a given network, will be:

$$\pi_{\mathrm{c}}^{\mathrm{i}} = \frac{F_c}{\dot{E}_c^{i-1}} - C_c^i$$

where $\dot{\mathrm{E}}_{\mathrm{c}}^{\mathrm{i-1}}$ is the maximum efficiency level observed in network c, till period $i-1$.

Therefore, the present value of the firm's profit is:

$$\pi = \sum_{\mathrm{i}=1}^{\infty} \delta^{\mathrm{i}} \left[\sum_{c=1}^{n} \left(\frac{F_c}{\dot{E}_c^{i-1}} - C_c^i \right) \right]$$

The company maximizes the present value of profits, subject to the constraints:

$$C_j^i \geq \bar{C}_j^i, \qquad \forall\, j = 1,2,\dots,n$$

The Lagrangean is:

$$\mathcal{L} = \sum_{\mathrm{i}=1}^{\infty} \delta^{\mathrm{i}} \left[\sum_{c=1}^{n} \left(\frac{F_c}{E_c^{i-1}} - C_c^i \right) \right] + \sum_{\mathrm{i}=1}^{\infty} \sum_{c=1}^{n} \left[\lambda_{ic} \left(C_c^i - \bar{C}_c^i \right) \right]$$

Since, by definition,

$$E_c^{i-1} = \frac{F_c}{C_c^{i-1}}$$

The Lagrangean can be written as follows:

$$\mathcal{L} = \sum_{\mathrm{i}=1}^{\infty} \delta^{\mathrm{i}} \left[\sum_{c=1}^{n} \left(C_c^{i-1} - C_c^i \right) \right] + \sum_{\mathrm{i}=1}^{\infty} \sum_{c=1}^{n} \left[\lambda_{ic} \left(C_c^i - \bar{C}_c^i \right) \right]$$

As had happened before, this Lagrangean is not correct, in that it assumes that the maximum efficiency level attained by network c in the past is the efficiency level attained by that network in the previous period. To be precise, the Lagrangean is:

$$\mathcal{L} = \sum_{\mathrm{i}=1}^{\infty} \delta^{\mathrm{i}} \left[\sum_{c=1}^{n} \left(\frac{F_c}{max\{E_c^0; E_c^1; \dots; E_c^{i-1}\}} - C_c^i \right) \right] + \sum_{\mathrm{i}=1}^{\infty} \sum_{c=1}^{n} \left(C_c^i - \bar{C}_c^i \right)$$

Thus, the solution of the first Lagrangean will only be the solution of the "correct" Lagrangean if, in the solution, the efficiency level does not decrease over time, for all the networks.

The Kuhn-Tucker conditions are:

$$\frac{\partial \mathcal{L}}{\partial C_c^i} = 0, \qquad c = 1,2, \dots, n$$

$$\frac{\partial \mathcal{L}}{\partial \lambda_{ic}} \geq 0, \qquad c = 1,2, \dots, n$$

$$\lambda_{ic} \geq 0, \qquad c = 1,2, \dots, n$$

$$\lambda_{ic} \frac{\partial \mathcal{L}}{\partial \lambda_{ic}} = 0, \qquad c = 1,2, \dots, n$$

Solving the system, one obtains:

$$\frac{\partial \mathcal{L}}{\partial C_c^i} = 0 <=> \lambda_i = \delta^i - \delta^{i+1}$$

One can conclude that $\lambda_{ic} > 0$ and, therefore:

$$\lambda_{ic} \frac{\partial \mathcal{L}}{\partial \lambda_{ic}} = 0 <=> C_c^i = \bar{C}_c^i$$

Since, by assumption, $\bar{C}_c^{i+1} \geq \bar{C}_c^i$, one can conclude that $C_c^{i+1} \geq C_c^i$, that is, the efficiency level chosen by the company, for each network, does not decrease over time. Thus, this solution is also the solution of the "correct" Lagrangean.

Hence, one can conclude that this model incentivizes the company to maximize efficiency in all of its networks.

The only flaw of this model is that, apparently, it allows profits to the company whenever there is technological progress. Nevertheless, the regulator defines, in each regulatory period, that the allowed revenue, for each network, should be a percentage lower than the cost the network would face if it operated with the maximum efficiency level attained in the past. This percentage should reflect the technological progress and should not be calculated based on the company's performance. Thus, the allowed revenue for each network, in each period, will be:

$$R_c^i = min\{C_c^0; C_c^1; \dots; C_c^{i-1}\} * (1 - X_c^i)$$

where X_c^i is the predicted percentage of the cost reduction permitted by the technological progress, in each network.

Thus, the company's profit, in each network, is:

$$\pi = (1 - X_c^i)C_c^{i-1} - C_c^i$$

Since the company is incentivized to produce, in every period, with minimum cost, the company's profit, for each network, can be written as follows:

$$\pi = (1 - X_c^i)\bar{C}_c^{i-1} - \bar{C}_c^i$$

In order to allow zero profit to the company, the regulator would set X_c^i as follows:

$$\pi = 0 <=> X_c^i = \frac{\bar{C}_c^{i-1} - \bar{C}_c^i}{\bar{C}_c^{i-1}}$$

It is noteworthy that, during periods when no technological progress is expected, i.e., $\bar{C}_c^i = \bar{C}_c^{i-1}$, the technological parameter is set to zero.

The consumer price will be the ratio between the total allowed revenue and the amount of energy distributed, i.e.:

$$P^i = \frac{\sum_{c=1}^n R_c^i}{\sum_{c=1}^n Q_c}$$

Or, in other words:

$$P^i = \frac{\sum_{c=1}^{n}\left[min\left\{C_c^0; C_c^1; \dots; C_c^{i-1}\right\} * \left(1 - X_c^i\right)\right]}{\sum_{c=1}^{n} Q_c}$$

Thus, this model not only incentivizes the company to maximize efficiency in all of its networks, but also offers zero profit to the company, giving all the advantages of technological progress to consumers.

1.5 Obstacles to the Implementation of the New Proposal

Although it is clearly better, this new proposal cannot be used yet. In fact, contrary to what was assumed in the analysis, the amount of energy that the company has to distribute is not constant. Thus, in order to apply the proposed model, the regulator needs to estimate a cost function for each of the networks. However, the regulator has not sufficient data to estimate a cost function for each network. This reason led the regulator to choose to use all available observations to construct the cost function for the company as a whole, without differentiating the networks. As networks are, indeed, different, this approach does not seem appropriate. In the future, when the regulator is in possession of a sufficient number of observations, the estimation of a cost function for each network should take place and the proposed model should be used. However, since, given the current regulatory model, the company has an incentive not to be efficient (as showed in section I), the cost functions that the regulator will estimate in the future will be biased. Thus, I propose a new regulatory model to be used until there are enough observations to estimate a cost function for each network. This proposed regulatory model should incentivize the company to minimize costs in all networks, so that the resulting observations allow a correct estimation of the cost function.

The Model:

As mentioned, this new model aims to incentivize the company to maximize efficiency, so that the regulator can know the firm's cost function to later apply the model described in section 1.4.

Thus, there is a short time horizon for the application of this model. I consider that, during this time horizon, the exogenous factors that influence the cost function (such as the extent of the network, the proportion of the Low Voltage network, the proportion of energy distributed in Low Voltage, the client concentration, etc.) are constant, for any network. Hence, the only variable is the amount of energy to distribute.

This model assumes that unit costs do not increase with the quantity produced. Thus, if a given network was able to produce a given quantity with a given unit cost, the minimum unit cost required for the same network to produce a higher quantity is not greater than the referred unit cost.

The model also assumes that the total cost does not decrease with the amount of energy to distribute. Thus, if a given network produced a given quantity with a given total cost, the minimum total cost required for the same network to produce a lower quantity is not greater than the referred total cost.

Thus, in this model, the allowed revenue is, in each period, for each network, the minimum between:

a) the product of the amount of energy that the network has to distribute and the minimum unit cost obtained by the network to produce any lower amount

b) the minimum total cost obtained by the network to distribute any higher (or equal) amount

<u>Note:</u> If the network has never distributed a lower amount than the one it is required to distribute, a) is set to $+\infty$. Likewise, if the network has never distributed a higher amount than the one it is required to distribute, b) is set to $+\infty$.

Notice that, in order to determine both the minimum unit cost obtained by the network to produce any lower amount and the minimum total cost obtained by the company to distribute any higher (or equal) amount of energy, should only be used the data that was available at the beginning of the implementation of the model. Otherwise, the company would have incentives not to minimize its costs, since maximizing efficiency would lead to a decrease in future revenues.

The network's profit will be the minimum between:

- an (exogenous) proportion of the difference between the allowed revenue and the obtained cost

- the difference between the allowed revenue and the obtained cost

In order to prevent the company from having excessive profits, it is established that the company only profits a proportion of the difference between its revenue and its costs. The remainder of the difference between the network's revenue and the network's cost is returned to customers in the form of lower tariffs for the following period. The proportion of the difference between revenue and costs that is absorbed by the company is set so that the company has zero economic profit, that is, the company will only profit the return on capital invested at the rate of its cost of capital.

I will now analyze whether the proposed regulatory model incentivizes the company to be efficient, i.e., to minimize its costs.

Let

q be the amount of energy required for a given network to distribute

$\overline{c}$ be the minimum unit cost that the network has obtained producing a lower amount than q

c be the unit cost obtained by the network to produce q

c^{-} be the minimum cost required for the network to produce q. By definition:

$$c^{-} \leq \overline{c}$$

T be the minimum total cost obtained by the network to distribute any quantity not lower than q

q_{+} be the quantity distributed with the cost T

R be the revenue, defined by the regulator as follows:

$$R = \min\,(q\overline{c}; T)$$

γ be the (exogenous) proportion of the difference between the network's revenue and its cost that will be absorbed by the firm

$$0 < \gamma \leq 1$$

rem be the remuneration of the capital invested in the network, at the rate of the firm's cost of capital

η be the estimated cost elasticity of the network

π be the network's profit, defined as follows:

$$\pi = \min(\gamma[R - qc]; R - qc)$$

The company will maximize the network's profit, subject to the constraint:

$$c \geq c^{-}$$

Now, there are 2 scenarios:

Scenario 1: $q\bar{c} \leq T$

Scenario 2: $q\bar{c} > T$

I will study the company's behavior under both scenarios.

Scenario 1: $\mathbf{q\bar{c} \leq T}$

$$R = q\bar{c}$$

$$\pi = \min(\gamma[q\bar{c} - qc]; q\bar{c} - qc)$$

Hence, the network's profit can be defined as:

$$\pi = \gamma(q\bar{c} - qc), \qquad \forall\, c \leq \bar{c}$$

$$\pi = (q\bar{c} - qc), \qquad \forall\, c > \bar{c}$$

It's straightforward to see that the company will choose $c \leq \bar{c}$. Thus, the company will maximize:

$$\pi = \gamma(q\bar{c} - qc)$$

Subject to the following constraints:

$$c \leq \bar{c}$$

$$c \geq c^{-}$$

The solution is:

$$c = c^{-}$$

One can conclude that, under this scenario, the company will minimize costs.

The expected costs are:

$$E(\text{costs}) = \bar{c}[(q - \bar{q})\eta + \bar{q}]$$

γ is set so that

$$\gamma(q\bar{c} - \bar{c}[(q - \bar{q})\eta + \bar{q}]) = \text{rem}$$

Which implies

$$\gamma = \frac{\text{rem}}{\bar{c}(q - \bar{q})(1 - \eta)}$$

If the company can't recover its cost of capital, i.e. $\text{rem} > \bar{c}(q - \bar{q})(1 - \eta)$, the allowed revenue will be defined as

$$R = E(\text{costs}) + \text{rem}$$

$$R = \bar{c}[(q - \bar{q})\eta + \bar{q}] + \text{rem}$$

Scenario 2: $q\bar{c} > T$

$$R = T$$

$$\pi = \min(\gamma[T - qc]; T - qc)$$

Hence, the network's profit can be defined as:

$$\pi = \gamma(T - qc), \qquad \forall\, c \leq \frac{T}{q}$$

$$\pi = (T - qc), \qquad \forall\, c > \frac{T}{q}$$

It's straightforward to see that the company will choose $c \leq \frac{T}{q}$. Thus, the company will maximize:

$$\pi = \gamma(T - qc)$$

Subject to the following constraints:

$$c \leq \frac{T}{q}$$

$$c \geq c^{-}$$

The solution is:

$$c = c^{-}$$

One can conclude that the company will also minimize costs under this scenario.

The expected costs are:

$$\mathrm{E(costs)} = \mathrm{T}\left[\left(\frac{\mathrm{q}}{\mathrm{q}_{+}} - 1\right)\eta + 1\right]$$

γ is set so that

$$\gamma\left(\mathrm{T} - \mathrm{T}\left[\left(\frac{\mathrm{q}}{\mathrm{q}_{+}} - 1\right)\eta + 1\right]\right) = \mathrm{rem}$$

Which implies:

$$\gamma = \frac{\mathrm{rem}}{\mathrm{T}\left(1 - \frac{\mathrm{q}}{\mathrm{q}_{+}}\right)\eta}$$

If the company can't recover its cost of capital, i.e. $\mathrm{rem} > T\left(1 - \frac{\mathrm{q}}{\mathrm{q}_{+}}\right)\eta$, the allowed revenue will be defined as

$$\mathrm{R} = \mathrm{E(costs)} + \mathrm{rem}$$

$$\mathrm{R} = \mathrm{T}\left[\left(\frac{\mathrm{q}}{\mathrm{q}_{+}} - 1\right)\eta + 1\right] + \mathrm{rem}$$

It's noteworthy that, under both scenarios, γ is exogenous, in the sense that it does not depend on the company's behavior.

Thus, the company will minimize its costs in all its networks. Furthermore, the company will make zero economic profit.

SECTION 2 – ANALYSIS OF THE IMPLEMENTATION OF NEW PRODUCTION PROCESSES

2.1 Brief Critic of the Current Regulatory Model

Whenever a new production process that allows the company to produce at lower costs is available, it will only be implemented at the most convenient time for the firm. Ideally, the company would put into practice the production process that allows a cost reduction at the time it is available. However, the current Portuguese regulatory model may encourage the company to postpone the application of the new production process.

The current Portuguese regulatory model establishes, at the beginning of each regulatory period, a maximum revenue for the regulated company. This allowed revenue is calculated based on the company's performance in the past. Specifically, the revenue set by the regulator allows the company to break even when the firm's efficiency, as a whole, during the regulatory period, is equal to the highest efficiency level attained by any of the networks in any previous year. Thus, if a company discovers a new production process, that permits cost reduction, in the last year of a regulatory period, it may be optimum for the firm to postpone the implementation of this new process to the beginning of the following regulatory period. In fact, if the company applies the production process in the last year of the regulatory period, it will profit, on that year, the cost variation allowed by the new production process. In subsequent regulatory periods, revenue will be set taking into account the existence of the new production process and, consequently, company will no longer profit by using this new process. If the company waits for the beginning of the next regulatory period to implement the new production process, the revenue set by the regulator would not take the new production process into account (since the regulator wouldn't know about the new production process) and, consequently, the company would profit the cost variation allowed by the new production process in all the years of the new regulatory period.

The loser in this "game" is, of course, society, as a whole. By failing to implement a production process that permits cost reduction when it's available, the company is being inefficient. In this section, I investigate the present value of society's loss. I also present a proposal for action for the regulator that eliminates the social losses resulting from the postponement of the implementation of new production processes, by the regulated firm.

2.2 Analysis of the Company's Behavior

Let

D be the regulatory period where the new production process is discovered

M be the regulatory period where the new production process is implemented

p be the year of the regulatory period where the new production process is first available

v be the year of the regulatory period where the new production process is implemented

δ be the annual discount rate, so that:

$$\delta = \frac{1}{1+r}$$

Where r is the annual interest rate.

t is the number of years in a regulatory period

Therefore,

δ^t is the discount rate between regulatory periods

Let also

ΔC be the annual cost variation permitted by the new production process

$R_c^{i,j}$ be the allowed revenue for year j of the regulatory period i, in network c

$C_c^{i,j}$ be the observed cost for year j of the regulatory period i, in network c

n be the number of networks

The company's profit is:

$$\pi = \sum_{j=0}^{\infty} (\delta^t)^j \left[\int_{i=0}^{t} \delta^i \left[\sum_{c=1}^{n} \left(R_c^{j,t} - C_c^{j,t} \right) \right] \right]$$

The company maximizes profits, subject to one of the following constraints:

1) $M > D$
2) $M = D \cap v \geq p$

That is, the company can apply the new production process in any regulatory period after the one when the process was discovered (constraint 1) or, alternatively, the company can apply the new production process in the regulatory period that the process was discovered. In this case, the company cannot implement the new production process in a year before the year that the process is available (constraint 2).

For simplicity, I will consider the following assumption:

Assumption 1: The maximum revenue allowed by the regulator is, indeed, the minimum cost that the company can attain when it does not implement any new production process that allows cost reduction.

Hence,

$R_c^{j,t} = C_c^{j,t}, \quad \forall\, j \neq M$

$R_c^{j,t} = C_c^{j,t}, \quad j = M \cap t < v$

$R_c^{j,t} = C_c^{j,t} + \Delta C, \quad j = M \cap t \geq v$

Given the assumption, the company's profit function can be written as follows:

$$\pi = (\delta^t)^M \left[\int_{i=v}^{t} \delta^i [\Delta C]\right]$$

A company may choose one of two scenarios: apply the production process in the regulatory period when it is available (using, in this case, constraint 2) or apply the production process in any regulatory period after the one when the production process was discovered (using, in this case, constraint 1).

Therefore, I will analyze the company's profit, in both cases, to know the decision of a company that acts in order to maximize profits.

If the company decides to implement the new production process in any regulatory period after the period in which it is available, it will face the following problem:

$$Max \quad \pi = (\delta^t)^M \left[\int_{i=v}^{t} \delta^i [\Delta C]\right]$$

s.t

$$M > D$$

The derivative of the profit (π), with respect to the regulatory period in which the production process is implement (M) is:

$$\frac{\partial \pi}{\partial M} = (\delta^t)^M \ln(\delta^t) \int_{i=v}^{t} \delta^i [\Delta C]$$

δ is, by definition, less than 1 and t is, by definition, strictly positive. Thus, $\ln(\delta^t)$ is strictly negative. Hence, since both $(\delta^t)^M$ and $\int_{i=v}^{t} \delta^i[\Delta C]$ are positive, one can conclude that $\frac{\partial \pi}{\partial M} < 0$. Therefore, in this scenario, the company will choose the lowest possible value for M. Since M is a discrete variable and the company faces the constraint $M > D$, the firm will choose $M = D + 1$. Thus, if the company chooses not to implement the new production process in the regulatory period when it is available, it will implement it in the subsequent regulatory period.

Replacing $M = D + 1$ in the firm's profit, one obtains:

$$\pi = (\delta^t)^{D+1} \left[\int_{i=v}^{t} \delta^i[\Delta C] \right]$$

The company chooses v in order to maximize profits.

$$\frac{\partial \pi}{\partial v} = (\delta^t)^{D+1}(-\delta^v[\Delta C]) = -\delta^{tD+t+v}[\Delta C]$$

The derivative of profits (π) with respect to the year in which the firm implements the new production process (v) is strictly negative. Thus, the company will choose the lowest possible value for v, i.e., $v = 0$.

Hence, the company's profit, if the new production process is not implemented in the regulatory period when it was available, is:

$$\pi = (\delta^t)^{D+1} \left[\int_{i=0}^{t} \delta^i[\Delta C] \right]$$

If the company implements the new production process in the regulatory period in which it is available, it will face the following problem:

$$Max \quad \pi = (\delta^t)^D \left[\int_{i=v}^{t} \delta^i [\Delta C] \right]$$

s.t

$$v \geq p$$

The Lagrangean is:

$$\mathcal{L} = (\delta^t)^D \left[\int_{i=v}^{t} \delta^i [\Delta C] \right] + \lambda(v - p)$$

The Kuhn-Tucker conditions are:

$$\frac{\partial \mathcal{L}}{\partial v} = 0$$

$$\frac{\partial \mathcal{L}}{\partial \lambda} \geq 0$$

$$\lambda \geq 0$$

$$\lambda \frac{\partial \mathcal{L}}{\partial \lambda} = 0$$

Solving the system, one obtains:

$$\frac{\partial \mathcal{L}}{\partial v} = 0 <=> \lambda = \delta^{tD+v} \Delta C$$

Since $\lambda > 0$:

$$\lambda \frac{\partial \mathcal{L}}{\partial \lambda} = 0 <=> \frac{\partial \mathcal{L}}{\partial \lambda} = 0 <=> v = p$$

Thus, one concludes that if the company opts to implement the new production process in the regulatory period when it is available $(M = D)$, the firm will implement it in the moment that it is available as well $(v = p)$. Thus, the company's profit is:

$$\pi = (\delta^t)^D \left[\int_{i=p}^{t} \delta^i [\Delta C] \right]$$

Hence, the company will opt not to implement the new production process in the moment that it is available, preferring to wait for the beginning of the following regulatory period, if (and only if):

$$(\delta^t)^{D+1} \left[\int_{i=0}^{t} \delta^i [\Delta C] \right] > (\delta^t)^D \left[\int_{i=p}^{t} \delta^i [\Delta C] \right] <=> \delta^t > \frac{\int_{i=p}^{t} \delta^i [\Delta C]}{\int_{i=0}^{t} \delta^i [\Delta C]} <=>$$

$$<=> \delta^t > \frac{\int_{i=p}^{t} \delta^i}{\int_{i=0}^{t} \delta^i} <=> \delta^t > \frac{\delta^t - \delta^p}{\delta^t - 1} <=> \delta^t(\delta^t - 1) < \delta^t - \delta^p <=>$$

$$<=> \delta^p < \delta^t(2 - \delta^t) <=> p \ln(\delta) < \ln[\delta^t(2 - \delta^t)] <=> p > \frac{\ln[\delta^t(2 - \delta^t)]}{\ln(\delta)} <=>$$

$$<=> p > \frac{\ln(\delta^t) + \ln(2 - \delta^t)}{\ln(\delta)} <=> p > t + \frac{\ln(2 - \delta^t)}{\ln(\delta)}$$

Note that $0 < \delta < 1$, $\ln(2 - \delta^t) > 0$ and $\ln(\delta) < 0$, so $\frac{\ln(2-\delta^t)}{\ln(\delta)} < 0$. Thus, for any values of the discount rate (δ) and length of the regulatory period (t), there is always a time lag where should the company discover a new production process that reduce costs, the firm will not apply it immediately, preferring to wait for the start of the next regulatory period.

The value of the loss to society, that is, the sum, from the moment the company discovers the new process until the firm implements it, of the difference between the cost faced by the company and the cost that the company would face if the new process was implemented, will be, evidently, a function of the period in which the new production process is available (p). As long as $p < t + \frac{\ln(2-\delta^t)}{\ln(\delta)}$, there is no society's loss, in the sense that, in that case, the company implements the new production process in the moment that it is available. For other values of p, it is straightforward to see that the society's loss will be greater the lower p. In fact, the value of the loss to society is defined as the sum, from the moment the company discovers the new process until the firm implements it, of the difference between the cost faced by the company and the cost that the company would face if the new process was implemented. Since, for $\mathrm{p} > t + \frac{\ln(2-\delta^t)}{\ln(\delta)}$, the moment when the new production process is implemented does not depend on p (it will always be the beginning of the following regulatory period) and the difference between the cost faced by the company and the cost that the company would face if applying the new production process does not depend on p as well, one can conclude that, when the company is incentivized to wait for the next regulatory period to apply the new production process, the later the new production process has been discovered, the lower the loss to society.

As stated above, the value of the loss to society, for a given p, is:

$$\int_{i=p}^{t} \delta^i \Delta C\,, \qquad for \qquad p > t + \frac{\ln(2-\delta^t)}{\ln(\delta)}$$

$$0, \qquad\qquad for \qquad p \leq t + \frac{\ln(2-\delta^t)}{\ln(\delta)}$$

I will now analyze the present value of the loss to society, that is, the sum, for all values of p, of the loss to society for a given p multiplied by the probability of p occur. Thus, the present value of the loss to society is, for each regulatory period:

$$VP = \int_{p=0}^{t} P(p) f(p)$$

where VP is the present value of the loss to society for each regulatory period, $P(p)$ is the probability that p occurs and $f(p)$ is the function of the value of the loss to society, for a given p, defined above. Replacing $f(p)$, one gets:

$$VP = \int_{p=0}^{t+\frac{\ln(2-\delta^t)}{\ln(\delta)}} P(p)0 + \int_{p=t+\frac{\ln(2-\delta^t)}{\ln(\delta)}}^{t} P(p) \int_{i=p}^{t} \delta^i \Delta C$$

Simplifying:

$$VP = \Delta C \int_{p=t+\frac{\ln(2-\delta^t)}{\ln(\delta)}}^{t} P(p) \int_{i=p}^{t} \delta^i$$

For simplicity, I will consider the following assumption:

Assumption 2: p is a random variable uniformly distributed between 0 and t, i.e., the probability of a new production process to be discovered in a given moment is equal to the probability of the new production process to be discovered in any other moment. This hypothesis does not seem very restrictive, in that the effort made by the company to find new ways of cost reduction does not depend on the time when such effort occur.

Thus, by assumption 2,

$$P(a) = P(b), \quad \forall \; a, b \in [0, t]$$

Moreover,

$$\int_{p=0}^{t} P(p) = P$$

where P is the probability of a new production process that allows the company to reduce its costs to be discovered in a given regulatory period.

One concludes, then, that $P(p)$ is a constant, that is, does not depend on p. Hence,

$$\int_{p=0}^{t} P(p) = P <=> P(p) = \frac{P}{t}$$

The present value of society's loss, for each regulatory period, can be simplified. In fact:

$$VP = \frac{P}{t}\Delta C \int_{p=t+\frac{\ln(2-\delta^t)}{\ln(\delta)}}^{t} \int_{i=p}^{t} \delta^i = \frac{P}{t}\Delta C \int_{p=t+\frac{\ln(2-\delta^t)}{\ln(\delta)}}^{t} \frac{\delta^t - \delta^p}{\ln(\delta)} =$$

$$= \frac{P}{t}\Delta C \left[\frac{\delta^t p}{\ln(\delta)} - \frac{\delta^p}{[\ln(\delta)]^2}\right]_{t+\frac{\ln(2-\delta^t)}{\ln(\delta)}}^{t} =$$

$$= \frac{P}{t}\Delta C \left[\frac{\delta^t t}{\ln(\delta)} - \frac{\delta^t}{[\ln(\delta)]^2} - \frac{\delta^t\left(t + \frac{\ln(2-\delta^t)}{\ln(\delta)}\right)}{\ln(\delta)} + \frac{\delta^{\left(t+\frac{\ln(2-\delta^t)}{\ln(\delta)}\right)}}{[\ln(\delta)]^2}\right] =$$

$$= \frac{P}{t}\Delta C \left[-\frac{\delta^t}{[\ln(\delta)]^2} - \frac{\delta^t \frac{\ln(2-\delta^t)}{\ln(\delta)}}{\ln(\delta)} + \frac{\delta^{\left(t+\frac{\ln(2-\delta^t)}{\ln(\delta)}\right)}}{[\ln(\delta)]^2}\right] =$$

$$= \frac{P}{t}\Delta C \left[\frac{\delta^t}{[\ln(\delta)]^2}\left(\delta^{\frac{\ln(2-\delta^t)}{\ln(\delta)}} - 1 - \ln(2-\delta^t)\right)\right] = \frac{P}{t}\Delta C \left[\frac{\delta^t}{[\ln(\delta)]^2}\left((1-\delta^t) - \ln(2-\delta^t)\right)\right]$$

The Expected Present Value of society's loss is the discounted sum of society's loss for each regulatory period, i.e.:

$$VAP = \sum_{j=1}^{\infty} \delta^{tj} \frac{P}{t}\Delta C \left[\frac{\delta^t}{[\ln(\delta)]^2}\left((1-\delta^t) - \ln(2-\delta^t)\right)\right] =$$

$$= \frac{\delta^t}{1-\delta^t}\frac{P}{t}\Delta C \left[\frac{\delta^t}{[\ln(\delta)]^2}\left((1-\delta^t) - \ln(2-\delta^t)\right)\right] = \frac{P\Delta C}{[\ln(\delta)]^2}\frac{\delta^{2t}[(1-\delta^t) - \ln(2-\delta^t)]}{(1-\delta^t)t}$$

The current Portuguese regulatory model leads to society's losses, because the company is not always incentivized to use the new production processes when they are available. I will suggest a proposal for action for the regulator to eliminate these losses and, consequently, to improve society's welfare.

2.3 Proposal for Action

The company only has an incentive to wait for the beginning of the following regulatory period to apply a new production process discovered in a given period because the firm knows that, if it implement the new process when it is available, the allowed revenue for the following regulatory periods will decrease. Therefore, the guarantee by the regulator that the revenue in the next regulatory period is not affected by the introduction of new production processes during a given regulatory period could, apparently, eliminate the incentive of the company to postpone the implementation of a new production process. However, this solution would only strengthen the company's incentive to postpone the implementation of a new production process. But instead of starting the implementation of a new process at the beginning of the new regulatory period, the firm would begin the implementation of these processes an infinitesimal time after the start of the regulatory period, profiting, thus, the variation in costs in two regulatory periods. Nevertheless, this can be remedied by the regulator if he establishes that if the implementation of the new process is too near the beginning of the regulatory period, the revenue of the following period will be affected. Thus, the regulator would state that the application of new production processes will not impact the revenue of the following regulatory period if and only if $v > t + \frac{\ln(2-\delta^t)}{\ln(\delta)}$, being v the moment in the regulatory period where the new process is implemented, t the length of the regulatory period and δ the firm's discount rate. However, this solution, by itself, is not enough, since if the company discovered a new production process at a time immediately before $t + \frac{\ln(2-\delta^t)}{\ln(\delta)}$, it would be incentivized to postpone the implementation of the production process, in order to benefit from higher revenue for the next regulatory period. Thus, in order to overcome this issue, the regulator must establish that, although the revenue for the next regulatory period is not affected by the implementation of a new process if $v > t + \frac{\ln(2-\delta^t)}{\ln(\delta)}$, the variation in the costs allowed by the new production process in the first regulatory period in which it is applied is passed on to consumers, in the form of reduced tariffs for the following period. Thus, if the company discovers a new production process in a moment p such that $p \geq t + \frac{\ln(2-\delta^t)}{\ln(\delta)}$, it would be indifferent between implementing the production process immediately and postponing its implementation to the beginning of the following regulatory period. If the company discovers a new production process in any other moment, i.e., $p < t + \frac{\ln(2-\delta^t)}{\ln(\delta)}$, it would implement it as soon as possible, in the sense that, in this case, the firm has no incentive to wait for the following regulatory period, as shown previously.

Since, if the firm discovers a new production process in a moment p such that $p \geq t + \frac{\ln(2-\delta^t)}{\ln(\delta)}$, the company is indifferent between implementing the process immediately and postponing its implementation to the beginning of the next regulatory period, the creation of a cost (even infinitesimal) for the company if it doesn't apply the new process when it is available is sufficient to incentivize the firm to implement a new production process when it is available. Thus, the regulator could establish a fine for the company if the firm is not using a known production process that allows the production with lower costs. Even though the likelihood of the regulator to discover that the firm is not using a known production process is small, this fine is sufficient to encourage the company to implement a new production process when it is available.

Thus, in this new model, the company is incentivized to implement a new production process when it is available.