Price Discrimination under Customer Recognition and Mergers*

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Abstract

This paper studies the interaction between horizontal mergers and price discrimination by endogenizing the merger formation process in the context of a repeated purchase model with two periods and three firms wherein firms may engage in Behaviour-Based Price Discrimination (BBPD). From a merger policy perspective, this paper’s main contribution is two-fold. First, it shows that when firms are allowed to price discriminate, the (unique) equilibrium merger gives rise to significant increases in profits for the merging firms (the ones with information to price-discriminate), but has no effect on the outsider firm’s profitability, thereby eliminating the so-called ‘free-riding problem’. Second, this equilibrium merger is shown to increase industry profits at the expense of consumers’ surplus, leaving total welfare unaffected. This then suggests that competition authorities should scrutinize with greater zeal mergers in industries where firms are expected to engage in BBPD.

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1 Introduction

The large body of previous literature on the effects of horizontal mergers on firms’ pricing policies has mainly focused on the balance between anticompetitive price (market power) effects and pro-competitive merger-related efficiency improvements.\(^1\) It should be noted, however, that market power and efficiencies are not the only important channels through which horizontal mergers can affect the pricing policy of a merged firm.

One factor that plays a key role in explaining merged firms’ pricing strategies is their information about individual consumers. In many industries, mergers can change both companies’ individualized customer information sets as well as the way they process this information. In particular, horizontal mergers can enable the consolidated firm to gather and exploit better databases on consumers’ individual characteristics (or purchase histories), thereby affecting the conditions of this merged firm to embark on discriminatory pricing schemes. A particularly interesting variant of price discrimination is the so-called behaviour-based price discrimination (henceforth BBPD),\(^2\) which occurs when firms have information about consumers’ past behaviour and use this information to offer different prices to consumers with different purchasing histories.\(^3\)

A relevant example is the Great Britain’s gas and electricity retail supply market for domestic and Small and Medium Enterprise (SME) consumers. This is a market where not only several mergers have occurred during the last decade, giving rise to an industry where there are today six large energy supply companies which collectively have over 99% of Great Britain market share, but also one where further mergers are likely to occur since EDF Group announced (in September 2008) an offer for the acquisition of the entire share of capital of British Energy. Moreover, a recent report by the Office of Gas and Electricity Markets (Ofgem (2008)), the regulator for Britain’s gas and electricity industries, has revealed that, in this industry: (i) a substantial fraction of consumers are ‘switchers’ in the sense that they constantly seek out for the best deal in the market;\(^4\) and (ii) suppliers are well aware of these consumers’ dynamics and do take them into account in their pricing decisions. In particular, “companies charge more to existing (“sticky”) customers whilst maintaining competitiveness in more price sensitive segments of the market. The ability to price differentially in this way means that pressure on prices in the most competitive segments of the market does not always constrain prices for all other consumers.” (Ofgem (2008), paragraph 1.21) In addition, according to a companion report by the same regulator, “vulnerable consumers are disproportionately affected by unjustified price differentials ... [and it is found] that lack of information, lack of internet access, reliance on cash and tight budgets all create barriers to switching

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\(^1\)See, for instance, Motta (2004, chapter 5) and Whinston (2006, chapter 3), for general discussions of the effects of horizontal mergers.


\(^3\)As pointed out by Fudenberg and Villas-Boas (2007), “[t]his sort of ‘behavior-based price discrimination’ (BBPD) and use of ‘customer recognition’ occurs in several markets, such as long-distance telecommunications, mobile telephone service, magazine or newspaper subscriptions, banking services, credit cards, labor markets; it may become increasingly prevalent with improvements in information technologies and the spread of e-commerce and digital rights management.” (p. 2) Along these lines, Gehrig and Stenbacka (2005) highlight that, “[a] typical example of behaviour-based price discrimination is a pricing scheme, which is contingent on the history of internet clicks.” (p. 132)

\(^4\)See paragraphs 1.4, 1.10 and 4.20 in Ofgem (2008).
for these consumers.” (Ofgem (2009), paragraph 6.1).

The recognition that the availability of individualized customer information can be improved through the process of mergers, on the one hand, and can affect the conditions for sustaining behaviour-based pricing schemes, on the other, raises a number of interesting questions. If price discrimination is permitted, what is the impact of BBPD on firms’ merger decisions? Does the ability of firms to engage in BBPD after a merger eliminate the so called free-riding problem identified by the previous mergers literature? What are the consumer and welfare effects of mergers when firms can engage in BBPD? Despite the empirical relevance of the interaction between horizontal mergers and price discrimination, the literature has devoted scarce attention to this topic.\(^5\) Hence, the answer to these and other related questions is not yet known.

This paper contributes to close this gap in the literature by endogenizing the merger formation process in the context of a repeated purchase model with two periods and three firms wherein firms may engage in BBPD. Consumers are assumed to be heterogeneous: some consumers are captive to a given firm and others are shoppers in the sense that they consider competing firms’ products as homogeneous and are, therefore, price-sensitive consumers. In the first period, firms cannot distinguish a captive consumer from a shopper (although the size of each customer segment is common knowledge to all firms in the industry). Thus, in the absence of purchase histories, oligopolists necessarily compete in uniform prices. In the second period, however, if price discrimination is permitted, firms can condition prices on observed purchase histories.\(^6\) In particular, they can differentiate between the prices they charge to customers with whom they have established a customer relationship and the prices by which they try to attract new consumers. We also assume that, in the beginning of the second period, i.e. before price competition takes place for the second time, a two-firm merger may occur allowing the merging parties to join their customer-information databases.

Within this theoretical framework, some interesting novel results are obtained. First, if firms are not allowed to merge but discrimination is permitted, then, in the second-period pricing game, all firms end up earning the same profit regardless of having or not gained access to the required information to engage in price discrimination.\(^7\) This is because, in the second period of the game, discriminating firms will compete very fiercely for shoppers and, as a result, end up not making extra profits in this segment of the market.\(^8\) This result should be compared with Esteves (2009), who, for the two-firm case, shows that price discrimination boosts both the discriminating and the non-discriminating firm’s second period profit. Hence, by relaxing the standard assumption that there are only two firms in the industry, the model proposed in this paper yields new economic

\(^5\) Two noteworthy exceptions are Reitzes and Levy (1995) and Cooper et. al (2005, Section III).

\(^6\) Put it another way, if permitted, price discrimination can only occur in the second period, when firms may have learnt consumers’ types by observing their first period choices.

\(^7\) The firm charging the lowest price in the first period of the game ends up selling not only to its captive consumers, but also to the entire group of shoppers. Hence, it learns nothing and is forced to sell at a single price in the second period of the game. In contrast, the remaining two firms sell only to their captive consumers in the first period and, therefore, by being able to recognize these captive consumers, will have the required information to engage in price discrimination in the subsequent period of the game.

\(^8\) When there are three or more firms in the market, then, in the second period, prices are set at the marginal cost level for shoppers whereas captive consumers are charged the monopoly (reservation) price.
insights which contrast with previous results in the literature. Second, if instead mergers are possible it follows that: (i) a merger will only occur in equilibrium in case price discrimination is permitted; (ii) the equilibrium merger configuration is unique; and (iii) the merger will involve the two firms with information to price discriminate in the second-period pricing game. In addition, even though, in equilibrium, this merger gives rise to significant increases in profits for the merging firms, the firm which is excluded from participation in the merger (the non-discriminating outsider firm) is not affected by the merger in terms of profits. This result then eliminates the so called ‘free-riding problem’ identified by the previous horizontal mergers literature (e.g. Salant, Switzer and Reynolds (1983)) and is, therefore, related to Reitzes and Levy (1995). These authors consider a spatial model where firms engage in perfect price discrimination (prices are contingent on consumer’s location) as a means of exploiting consumer diversity. Within that framework, the free-riding problem associated with mergers disappears since the merged firm continues to serve the same customers after the merger (and, hence, the outsider firms’ customers bases do not change with the merger either). Moreover, the outsiders’ pricing behaviour is unaffected by the merger. In contrast, in the present paper, in equilibrium, the merged firm is able to distinguish the types of all customers’ willing to buy from it and so it can capture some additional customers (namely, the shoppers who previously bought from the outsider firm). In addition, the merger does have an impact on both firms’ pricing strategies. If there is no merger, then, as mentioned above, all captive customers are charged the monopoly price whereas the customers willing to switch pay the marginal cost price. If instead a merger occurs in equilibrium, then while the merged firm has information about each customer’s type (and can, thus, engage in BBPD), the outsider has not. As a result, after the merger, the discriminating merged firm has an advantage over the outsider: the merged entity is able to entice some of the rival’s previous customers to switch, without damaging the profit from its locked in segment, whereas the outsider firm cannot protect its captive market from price cuts. This then softens the outsider firm’s pricing behaviour and boosts the merged firm’s profits from poached customers.

Lastly, and perhaps most importantly, we show that the equilibrium merger will increase industry profits at the expense of consumers’ surplus, leaving total welfare unaffected. Our results, thus, carry an important merger policy implication: irrespective of the welfare standard adopted by the competition authorities to appraise a proposed merger, they should scrutinize the mergers in industries wherein firms are expected to engage in BBPD with greater zeal.

This paper is mainly related to two strands in the literature. It is related to the literature on endogenous horizontal mergers since we explicitly model the merger formation process by making use

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9 As Chen (2005) highlighted in a report on the pros and cons of price discrimination, an important extension of the existing models of BBPD is to allow for more than two firms.

10 See also Rothschild et al. (2000) for a spatial model with price discrimination and firms choosing locations in anticipation of forming a merger, wherein the gains from merger participants exceed those of the excluded (outsider) firm.

11 It should be noted, however, that the adoption of the consumers’ welfare standard appears to be the current practice in the major antitrust jurisdictions. As Lyons (2002, p. 1) highlights, “most major competition authorities operate under legislation and guidelines that reject this [total surplus] standard, and no major competition authority seems to apply it consistently. Instead, they overwhelmingly focus on consumers, including industrial consumers, to the exclusion of the welfare of merging firms.” See also Pittman (2007).
of the coalition formation game which was first proposed by Hart and Kurz (1983). In particular, at the beginning of the second period, each of the three firms in the market simultaneously announces a list of players (including itself) that it wishes to form a coalition with. Firms that make exactly the same announcement then form a coalition together (i.e., merge).

The paper is also related to the stream of research on competitive BBPD where firms engage in price discrimination based on information about the consumers’ past purchases. Like other forms of price discrimination, BBPD can have antitrust and welfare implications. While in the switching cost approach purchase history discloses information about exogenous switching costs (e.g. Chen (1997) and Taylor (2003)), in the brand preference approach purchase history discloses information about a consumer’s exogenous brand preference for a firm (e.g. Villas-Boas (1999), Fudenberg and Tirole (2000)). A common finding in this literature is that BBPD tends to intensify competition and potentially benefit consumers.

Behaviour-based pricing tends to intensify competition and reduce profits in duopoly models where the market exhibits best response asymmetry,14 firms are symmetric and both have information to engage in BBPD (e.g. Shafer and Zhang (1995), Bester and Petrakis (1996), Chen (1997), Villas-Boas (1999), Fudenberg and Tirole (2000), Taylor (2003) and Esteves (2007)). There are, however, some models where firms can benefit from BBPD. This happens when firms are asymmetric (e.g. Shafer and Zhang (2000)), firms’ targetability is imperfect and asymmetric (Chen, et al. (2001)) and when only one of the two firms can recognize customers and price discriminate (Chen and Zhang (2009) and Esteves (2009)). The present paper will put forward that a change from two firms to three firms leads to qualitative differences in the economic outcomes derived and raises issues not covered in the literature so far. For antitrust policies, our analysis suggests that behaviour-based pricing can boost industry profit and harm consumers when a merger to duopoly is likely to occur.

The remainder of the paper is organized as follows. Section 2 lays out the formal framework. Section 3 presents two benchmark cases: (i) the case where price discrimination and mergers are not permitted; and (ii) the case where price discrimination is permitted while mergers are not allowed. Section 4 looks at mergers in industries where firms are expected to engage in BBPD. Price effects induced by a merger (with or without price discrimination being permitted) are studied in Section 5. Section 6 looks at welfare issues and Section 7 concludes.

## 2 The Model

Consider a market where $N = 3$ firms produce a product at a constant marginal cost which we normalize to zero without loss of generality. There are two periods, 1 and 2. The firms act to

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12 This model has, for instance, been applied by Vasconcelos (2006) to derive an upper bound to industry concentration in ‘endogenous sunk cost industries’ (Sutton (1991, 1998)).

13 Some other important contributions in this area are Gowrisankaran (1999), Kamien and Zang (1990), Fauli-Oller (2000) and Horn and Persson (2001), to name a few.

14 Following Corts (1998), the market exhibits best response asymmetry when one firm’s “strong” market is the other’s “weak” market. In BBPD models there is best-response asymmetry because each firm regards its previous clientele as its strong market and the rival’s previous customers as its weak market.
maximize their profits using a common discount factor \( \delta < 1 \). On the demand side of the market, we assume that the number of consumers is constant and normalized to one.\(^{15}\) Each consumer wishes to buy at most a single unit of the product in each period. To simplify the analysis we assume that consumers are naive in the sense that they do not anticipate any poaching attempt by firms in the future neither their incentives to merge. All consumers have an identical reservation value, \( v \).

Following Varian (1980), Narasimhan (1988), Chen and Zhang (2009) and Esteves (2009), we assume that consumers are heterogeneous. Some consumers are captive to a given firm (price-insensitive customers) and others are shoppers (price-sensitive customers). Captive consumers always purchase from their favorite firm as long as the quoted price is below \( v \) and cannot be poached by a rival firm.\(^{16}\) For shoppers, firms’ products are perfect substitutes: they always buy from the cheapest firm, as long as the price is not above \( v \). In a repeated purchase model, the price-sensitive customers might be willing to leave their previous supplier.\(^{17}\) Assume that the proportion of consumers captive to firm \( i \) is given by \( \gamma_i > 0 \) where \( i = 1, 2, 3 \). The total number of consumers who are captive to some firm is \( \gamma = \sum_{i=1}^{3} \gamma_i \). Thus, the number of shoppers is \( \beta = 1 - \gamma > 0 \). To simplify, we assume that the model is symmetric meaning that \( \gamma_1 = \gamma_2 = \gamma_3 = \gamma \), thus \( \beta = 1 - 3\gamma \). As we are interested in the case where \( \beta > 0 \), it follows that \( \gamma < \frac{1}{3} \).

2.1 Timing

In the first period, firms cannot distinguish a captive consumer from a shopper (although the size of each customer segment is common knowledge to all firms in the industry). Thus, in the absence of purchase histories, oligopolists necessarily compete in uniform prices. In the second period, however, firms may have learnt consumers’ types by observing their first period choices. So, if price discrimination is permitted, they can differentiate between the prices they charge to customers with whom they have established a customer relationship and the prices by which they try to attract new consumers. In particular, they may attempt to poach the rivals’ previous consumers, and at the same time extracting monopoly rents from captive clients. We also assume that, in the beginning of the second period, i.e. before price competition takes place for the second time, a two-firm merger may occur allowing the merging parties to join their customer-information databases.

\(^{15}\)This assumption fits well the motivating example discussed in the Introduction since, as pointed out in Ofgem (2009), “[i]n domestic energy supply markets the number of consumers is fixed (with the exception of new house builds) ... [and, hence,] changes in price differentials are unlikely to affect demand, given the scale identified. Furthermore, there is little potential here for new markets to be served through the ability of suppliers to price lower.” (paragraph 5.30).

\(^{16}\)Consumers may be fully captive due to a strong preference for a brand, high search or switching costs, or because the consumer has no information about the existence of other firms.

\(^{17}\)This is again in line with our motivating example. As stated in Ofgem (2009), “[a]s competition developed, the most price-elastic consumers switched away from the former incumbent to other suppliers. Though some consumers subsequently switched back, those consumers with the former incumbent supplier tend to be relatively price inelastic.” (paragraph 5.23)
2.2 The merger formation game

In order to determine the merger pattern, we make use of an endogenous merger model based on the coalition formation game which was first proposed by Hart and Kurz (1983). In particular, each firm \( i \in \{1, 2, 3\} \) simultaneously announces a list of players (including itself) that it wishes to form a coalition with. Firms that make *exactly* the same announcement then form a coalition together. For example, if firms 1 and 2 both announced coalition \( \{1, 2, 3\} \), while firm 3 announced something different (\( \{3\} \) or something else), then players 1 and 2 form a coalition.

In formal terms, firm \( i \)'s strategy is to choose a set of firms \( \hat{S}^i \), which is a subset of the set of firms in the industry \( \{1, 2, 3\} \) and includes firm \( i \). The set of strategies for firm \( i \) is, therefore, \( \Sigma^i = \left\{ \hat{S} \subset \{1, 2, 3\} \mid i \in \hat{S} \right\} \). Given firms’ announcements \( \alpha \equiv \left( \hat{S}^1, \hat{S}^2, \hat{S}^3 \right) \), the resulting coalition structure is \( C = \{C_1, \ldots, C_T\} \), where \( T \) denotes the number of *different* lists chosen by the 3 firms. \( C_i \cap C_j = \emptyset \) for \( i \neq j \) and \( \bigcup_{i=1}^T C_i = \{1, 2, 3\} \). Firms \( i \) and \( j \) belong to the same coalition \( C_k \) if and only if \( \hat{S}^i = \hat{S}^j \). Any side payments are ruled out with respect to membership decisions.

Two remarks are in order at this point. First, notice that \( \hat{S}^i \) (respectively, \( \hat{S}^j \)) is the *largest* set of firms firm \( i \) (respectively, firm \( j \)) would be willing to be associated with in the same coalition. As a result, the coalition \( C_k \) may in general be different from \( \hat{S}^i \) (respectively, \( \hat{S}^j \)). A coalition corresponds to an equivalent class, with respect to equality of strategies. Second, since in this paper we restrict attention to two-firm mergers, the resulting coalition structure \( C \) will be composed of at most three coalitions \( (T \in \{2, 3\}) \) and each coalition will be composed of at most two firms.

3 No-Merger Benchmark Cases

3.1 No price discrimination and no merger

We now investigate the case where mergers cannot occur and there is no price discrimination either because firms have no information (e.g. consumers behave anonymously) or because price discrimination is not permitted. It is straightforward to see that here the model is reduced to two replications of the static model. Consider first the static game. This setup is analogous to Narasimhan (1988) except that here we have three firms rather than two. It is then straightforward to prove the following result.\(^\text{18}\)

**Lemma 1** *There is no pure strategy equilibrium in prices.*

Note that firm \( i \) can always guarantee itself a profit equal to \( v_\gamma \) merely focusing on its captive customers. However, the presence of a positive fraction of shoppers creates a tension between its incentives to price low, in order to attract them, and to price high, in order to extract rents from its captive customers. This tension results in an equilibrium displaying price dispersion. There is a mixed strategy Nash equilibrium (henceforth, MSNE), the existence of which is proved by construction. Suppose that a symmetric mixed strategy involves firms charging a price no higher

\(^{18}\)For a similar proof see, for instance, Narasimhan (1988).
than $p$ with probability $F(p)$ with support $[p_{\text{min}}, v]$. If firm $i$ chooses price $p$ while the other firms use a mixed strategy, its expected profit is:

$$E\pi_i = p\gamma \left[ 1 - (1 - F(p))^2 \right] + p(\gamma + \beta) \left[ 1 - F(p) \right]^2 = p\gamma + p\beta [1 - F(p)]^2.$$  \hspace{1cm} (1)

In equilibrium, firm $i$ must be indifferent between quoting any price that belongs to the equilibrium support, where $p_i \in [p_{\text{min}}, v]$. Thus,

$$p\gamma + p\beta [1 - F(p)]^2 = v\gamma,$$

from which we obtain:

$$F(p) = 1 - \left( \frac{(v - p)\gamma}{p\beta} \right)^{\frac{1}{2}}.$$  \hspace{1cm} (2)

From the conditions which establish that $F(p_{\text{min}}) = 0$ it follows that $p_{\text{min}} = \frac{v\gamma}{\gamma + \beta}$.

In the two period game, overall expected profit for a representative firm is equal to

$$E(\pi) = (1 + \delta) \left\{ p\gamma + p\beta [1 - F(p)]^2 \right\}$$

Thus, it is straightforward to obtain the next proposition.

**Proposition 1** In the no-merger and no-discrimination benchmark case, there is a symmetric sub-game perfect MSNE where:

(i) each firm chooses a price randomly from the distribution function

$$F(p) = \left\{ \begin{array}{ll}
0 & \text{for } p < \frac{v\gamma}{\gamma + \beta} \\
1 - \left( \frac{(v - p)\gamma}{p\beta} \right)^{\frac{1}{2}} & \text{for } \frac{v\gamma}{\gamma + \beta} \leq p \leq v \\
1 & \text{for } p > v
\end{array} \right\}$$ \hspace{1cm} (3)

(ii) and each firm’s expected profit is equal to $E(\pi) = (1 + \delta) v\gamma$.

### 3.2 Price discrimination with no merger

Consider now the case where mergers cannot occur but price discrimination is permitted. As usual, we solve the game working back from the second period.
3.2.1 Second-period pricing game

In a repeated interaction, by collecting information about customers’ past behavior, a firm might be able to learn whether a consumer is a captive or a price-sensitive one who bought from a rival before. The fact that part of the market (of size $\beta$) did not buy from say firm 2 in period 1, allows firm 2 to infer that its first-period customers must be captive. When a firm achieves this type of learning, it may have incentives to entice those customers previously buying from a rival to switch over through later price discounts. Conversely, when a firm sells its product to all consumers who are willing to buy its product, it learns nothing and so cannot price discriminate in the subsequent period. One can then state the following lemma.

**Lemma 2** The cheapest firm (the high-market share firm) in period 1 will have no information to price discriminate in period 2. Only the firms selling exclusively to their captive customers in period 1 will have the required information to engage in price discrimination in the subsequent period.

The lowest-price firm in period 1 sells to the entire group of shoppers and to its captive consumers. Hence it learns nothing and is forced to set a single price in period 2. In contrast, the firms selling exclusively to their captive customers in period 1 are able to recognize these old customers and to infer that the non-purchasers are shoppers. This in turn implies that, in period 2, these (informed) firms will be able to engage in BBPD.

For a given price $p$ chosen by firm $i$ in period 1, firm $i$ is the lowest-price firm in period 1 (or the non-discriminating firm in period 2) with a probability equal to $\prod_{j \neq i} [1 - F_j(p)]$, where $F_j(p)$ denotes the probability that firm $j$’s price in period 1 is less than or equal to $p$. Hence, firm $i$ gets only its captive customers in period 1 (and can, thus, engage in price discriminating in period 2) with a probability equal to $1 - \prod_{j \neq i} [1 - F_j(p)]$.

With no loss of generality, suppose that in the first-period firms 1 and 2 sell only to their captive customers. Firm 3 is then the lowest-price firm in period 1, serving its captive market as well as the shoppers. (Firm 3’s demand is then equal to $\gamma + \beta = 1 - 2\gamma$). In this case, in the second period, firms 1 and 2 can recognize their old captive customers and the shoppers who bought from a rival before, and so they are able to price discriminate accordingly. Henceforth we will designate as *informed* a firm which before a merger takes place has a purchase history database which allows it to distinguish a captive from a switcher consumer in period 2. In contrast, an *uniformed* firm is the one that sold its product to both captive consumers and to switchers in period 1, and whose database before a merger does not allow the firm to distinguish a captive customer from one willing to switch. Let $p^c_i$ and $p^r_i$ denote the price set in the second period by firm $i$ $(i = 1, 2)$ to its own captive customers and to the rival’s previous customers, respectively.

**Proposition 2** The discriminating firms will charge their old captive customers the monopoly price $v$ and will charge the shoppers the marginal cost price. The non-discriminating firm charges its previous captive customers the monopoly price $v$.

The ability of firms 1 and 2 to fully separate their captive customers from shoppers that bought from a rival firm before, together with the incapability of the other firms to poach any of their
captive customers, allows firms 1 and 2 to charge their previous captive customers the reservation price, without fearing any poaching attempt by rival firms. Therefore, the discriminating firms’ profit from its old captive customers, denoted $\pi_i^0$, is equal to $\pi_i^0 = v\gamma$, $i = 1, 2$. However, as firms 1 and 2 compete for shoppers, in equilibrium, they set the marginal cost price to this group of consumers. Since consumers remain anonymous to the non-discriminating firm 3, this firm has no choice but to charge the same price to all consumers. When rival firms price at marginal cost to the shoppers, firm 3’s best response is to focus only on its captive customers and to quote them the monopoly price $v$.

Total second-period profit for a discriminating firm equals

$$\pi^d = \pi^0 + \pi^r = v\gamma,$$

whereas the second-period profit for a non-discriminating firm is equal to

$$\pi^{nd} = v\gamma.$$

Note that all firms earn the same profit in the second period of the game regardless of whether they have achieved the discriminating position or not. An interesting finding of the paper is then that when two firms acquire information about consumers and price discriminate in the second period, they compete fiercely for shoppers and end up making no additional profits in this segment. This is in sharp contrast with the results obtained in a duopoly model where just one firm achieves the discriminating position in the second period, in which price discrimination boosts both firms’ second-period profit (see Chen and Zhang (2009) and Esteves (2009)). Regarding consumers, price discrimination is good for shoppers (who pay the marginal cost price) and bad for captive consumers (who pay the monopoly price). It is worth remarking at this point that this result appears to be consistent with the experience of the Great Britain’s gas and electricity retail supply market for domestic and SME consumers, mentioned in the Introduction. Indeed, in this industry, and as highlighted in Ofgem (2008), the “Big 6 [suppliers] pursue a strategy of differential pricing, targeting the keenest, lowest margin prices at the most active part of the domestic market, while sustaining significantly higher prices for their less active customers.” (paragraph 6.14)\textsuperscript{19,20}

### 3.2.2 First-period pricing decisions

Consider next the equilibrium first-period pricing. Firms make their pricing choices simultaneously and rationally anticipating how such decisions will affect their profits in the subsequent period. As second period profits with discrimination are equal to second period profits with no discrimination,

\textsuperscript{19}See also paragraphs 1.21 and 8.25 in Ofgem (2008).

\textsuperscript{20}In Ofgem (2009), the regulator has “identified that those in social group D or E, those aged over 65, those without internet access and those who rent their accommodation (particularly if they do so from a private landlord) are less likely than others to switch supplier.” (paragraph 6.12).
price discrimination has no effect on first-period pricing decisions, as stated in the next proposition.

**Proposition 3** In the price discrimination and no-merger scenario, there is a symmetric subgame perfect MSNE where:

(i) in the first period each firm chooses a price randomly from the distribution function

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F(p) = \begin{cases} 
0 & \text{for } p < v \frac{\gamma}{\gamma + \beta} \\
1 - \left(\frac{(v-p)\gamma}{p\beta}\right)^{\frac{1}{2}} & \text{for } v \frac{\gamma}{\gamma + \beta} \leq p \leq v \\
1 & \text{for } p > v
\end{cases}
\]

(ii) and each firm expected profit is equal to \( E(\Pi) = (1 + \delta) v \gamma \).

The next corollary highlights that a change from two firms (as in the works by Chen and Zhang (2009) and Esteves (2009)) to (at least) three firms leads to important qualitative differences in the economic effects of BBPD in repeated markets where some consumers are captive while others are switchers.

**Corollary 1** In a market with \( n \geq 3 \) firms, if, at least, 2 firms are able to distinguish a shopper from a captive consumer and to price discriminate accordingly, then:

(i) all captive consumers pay the monopoly price \( v \) whereas shoppers pay the marginal cost price;

(ii) price discrimination has no effect on second period profit neither on first-period pricing choices.

Moving from two firms to three firms makes a substantial qualitative difference: when there are at least three firms, there are always at least two firms competing for any consumer willing to switch, and, therefore, these firms will bid away their profits from these new customers in an attempt to attract them. As a result, each discriminating firm charges shoppers the marginal cost price. The reason why three is the key number here is that with three (or more) firms, there are at least two firms competing to poach those customers willing to leave their current supplier.\(^{21}\)

The discriminating firms follow a strategy of “paying customers to switch” in an attempt to poach those customers willing to leave their current supplier while captive customers pay higher prices. In models where both firms have information to price discriminate and the market exhibits best-response asymmetry, a common finding is that (i) second-period prices tend to be all lower than if BBPD were not permitted; and (ii) new customers (shoppers) pay lower prices than repeated

\(^{21}\)By extending Chen’s (1997) model of BBPD in the switching cost approach to a triopoly market, Taylor (2003) shows that the results derived with, at least, three firms are different from those obtained in a duopoly market. In Chen’s duopoly model, the switchers have only one firm to switch to, so this other firm can make positive profits even on new customers, and the duopolists earn positive profits in equilibrium. Taylor (2003) shows, however, that with three or more firms, firms earn positive profits only on their current customers, and these rents are competed away in the first period.
(captive) customers. Here, in contrast, BBPD might not reduce all second period prices. While shoppers pay the marginal cost price, repeated locked in customers are charged their maximum willingness to pay.

4 Endogenous Merger Game

The literature on mergers has mainly focused on why firms merge and how they merge. The first question has been investigated assuming that the merger process is exogenous. In contrast, the second question has been modelled assuming that mergers are endogenous since each firm must decide whether or not (and with whom) to merge. This paper investigates the interaction between endogenous merger decisions and information-based price discrimination. When mergers are permitted in the beginning of the second period, i.e. before price competition takes place for the second time, a two-firm merger may occur allowing the merging parties to join their customer-information databases. Starting from an initial market with three independent symmetric firms, the model investigates which merger configuration is likely to emerge in equilibrium. For simplicity, we assume that a merger to monopoly would not be permitted by competition authorities.

4.1 Mergers with no discrimination

Consider first the case where a merger to duopoly is permitted while price discrimination practices are, for any reason, not allowed. If price discrimination is not permitted, then the post-merger game is similar to that in the benchmark case with no merger and no discrimination (Section 3.1), the only difference being that now, after the merger, we have two asymmetric firms rather than three symmetric firms.

Let us first look at the second-period price competition (after the merger). In the post-merger game, we have two asymmetric firms. The firm that results from the merger, say firm $M$, has a base of locked in customers equal to $2\gamma$, whereas the outsider firm, say firm $O$, has a group of locked in customers equal to $\gamma$. Both firms compete for customers willing to leave their current suppliers (i.e., the shoppers). Again, the presence of a positive fraction of price-sensitive customers creates a tension between firms’ incentives to price low, in order to attract them, and to price high, in order to extract rents from their captive customers. This tension results in an equilibrium displaying price dispersion.

**Proposition 4** In the post-merger game without price discrimination:

\[22\] After first period decisions have been made, each consumer that bought from firm $i$ will have a record in firm $i$’s database. If firm $i$ and $j$ decide to merge, they join their databases. In so doing, in a repeated interaction a consumer who has a record on one of the firm’s databases will be perceived as a captive old customer.
The merged firm \( M \) chooses a price randomly from the distribution:

\[
H_M(p) = \begin{cases} 
0 & \text{for } p < p_{M_{\text{min}}} \\
1 + \frac{\gamma}{\beta} - \frac{(\gamma+\beta)2v\gamma}{(\beta+2\gamma)p\beta} & \text{for } p_{M_{\text{min}}} \leq p \leq v \\
1 & \text{for } p > v 
\end{cases}
\]  

with support \([p_{M_{\text{min}}}, v]\) and a mass point at \( v \) equal to

\[
m = \frac{\gamma}{2\gamma + \beta}.
\]

where \( p_{M_{\text{min}}} = \frac{2v\gamma}{\beta+2\gamma} \).

The outsider firm \( O \) chooses a price randomly from the distribution:

\[
H_O(p) = \begin{cases} 
0 & \text{for } p < p_{M_{\text{min}}} \\
1 - \frac{2(\gamma(v-p))}{p\beta} & \text{for } p_{M_{\text{min}}} \leq p \leq v \\
1 & \text{for } p \geq v 
\end{cases}
\]

with support \([p_{M_{\text{min}}}, v]\).

(iii) The equilibrium profit for firm \( M \) equals

\[
\pi_M = 2v\gamma,
\]

and the profit for firm \( O \) equals

\[
\pi_O = \frac{2v\gamma}{\beta+2\gamma} (\beta + \gamma).
\]

**Proof.** See the Appendix. ■

In equilibrium, the merged firm uses a “Hi-Lo” pricing strategy. To squeeze more surplus from its captive customers, it charges the highest price \( v \), with probability \( m \). However, in order to attract new customers, it quotes occasionally a low price as a way to poach them from the rival. Note that the mass point \( m = \frac{\gamma}{2\gamma + \beta} \) is increasing in \( \gamma \) and decreasing in \( \beta \), highlighting that the merged company has more incentives to quote the monopoly price \( v \) when the group of locked in customers (price sensitive customers) is greater (smaller).

It is straightforward to see that \( H_M(p) < H_O(p) \), that is \( H_M(p) \) first-order stochastically dominates \( H_O(p) \). In other words, the merged company charges, on average, higher prices than the
outsider firm. When price discrimination is not permitted, a low price to entice the switchable customers damages the merged firm’s profit from its locked in customers. Thus, an increase in the group of the captive customers softens the merged firm and benefits the outsider firm which is able to increase its profit (given by \( \pi_O = 2vm(\beta + \gamma) \)).

**Corollary 2** When price discrimination cannot occur, the merger has no effect on the insiders’ second period profit but has a positive effect on the outsider firm’s profit.

As \( \pi_O = 2v\gamma (\beta + \gamma) / (\beta + 2\gamma) > v\gamma \), the outsider firm would benefit from a merger between its competitors. This result has been known in the literature as the free-riding effect of mergers (e.g. Salant, Switzer and Reynolds (1983) and Qiu and Zhou (2007)). However, with no discrimination, a merger has no effect on the insiders’ second period profit. This then implies that, even though a merger would allow firms to join their customer databases, as long as the merged entity cannot use these databases to price differently towards retained locked in customers and those willing to switch, no firm will embark on a merger in equilibrium. Each firm will announce a singleton coalition, and so, the resulting equilibrium coalition structure will be \( C = \{1\}, \{2\}, \{3\} \). As mergers do not occur in the beginning of the second period, the first period game is similar to the benchmark case with no mergers and no discrimination. Consequently, in period 1 firms behave as in Proposition 1.

### 4.2 Mergers with price discrimination

Let us now analyze mergers when price discrimination is possible. Two scenarios are relevant. In the first scenario, two informed firms are involved and the merger of customer purchase histories allows the merged entity to distinguish between an old (captive) customer from a customer that is willing to switch from his previous supplier (a price-sensitive customer). This will be the case when a merger occurs between those two firms that in period 1 gained the patronage of only their captive customers. In the second scenario, even though, in the second period, one of the merging firms would be able to distinguish a captive consumer from a switcher if it didn’t embark on a merger, when it merges with an uninformed firm, the merger of customer purchase databases will ‘obfuscate’ the merged entity in the sense that it will not be able to completely distinguish consumer types. An interesting point here is that the merger of customer databases will not always give the merged entity the information required to distinguish between its own captive customers and those that might be induced to switch.

#### 4.2.1 Second-period

**Both merging firms are informed** Suppose first that the merger is between two firms with information to distinguish whether a customer is a captive or a price sensitive one. The outsider firm (the lowest-price firm in period 1) cannot distinguish between its captive customers and those willing to switch in the second period. So, in this case, the merger not only increases the merged firm base of captive customers but also gives this merged firm an information advantage over the outsider to the merger. The merged entity has more flexibility in its pricing strategy because it can
charge two prices in the second period, one tailored at its old locked in customers \( p^o_M \) and the other tailored at the rival’s previous customers who might be willing to switch \( p^r_M \). The outsider, on the other hand, has no choice but to charge a uniform price in period 2 again. Let \( \hat{p}_N \) denote the non-discriminating firm’s second-period price.

**Lemma 3** The discriminating (merged) firm M will charge its old captive customers the monopoly price, namely \( p^o_M = v \).

Firm M is able to fully separate its segment of locked in customers (of size \( 2\gamma \)) from the segment composed of shoppers that bought from the outsider before and to price differently towards the two identified segments. As firm M does not compete with the outsider firm for its own captive customers, it charges them their maximum willingness to pay and its profit in this segment, denoted \( \pi^o_M \), will be equal to \( \pi^o_M = 2v \).

Look next at the second-period price competition for the segment of price sensitive customers who might be willing to switch from the outsider. Since consumers remain anonymous to the outsider firm, this firm is forced to quote the same price to all consumers.

**Proposition 5** There is no pure strategy equilibrium in prices for the group of consumers that bought from the outsider (non-discriminating) firm in period 1.

**Proof.** See the Appendix.

As in Esteves (2009) there is, however, an asymmetric MSNE. Let \( G_M(\hat{p}_N) \) denote the probability that the merged firm’s price to the rival’s previous customers is no higher than \( \hat{p}_N \) and \( \tilde{G}_N(p^r_M) \) denote the probability that the non-discriminating firm’s price is less than or equal to \( p^r_M \).

**Proposition 6** When the merged firm can engage in price discrimination, whilst the outsider firm cannot, price competition over the group of consumers (shoppers) previously buying from the outsider (non-discriminating) firm gives rise to an asymmetric MSNE in which:

(i) The non-discriminating firm chooses a price randomly from the distribution

\[
\tilde{G}_N(p^r_M) = \begin{cases} 
0 & \text{for } p^r_M < \hat{p}_{N\min} \\
1 - \frac{v\gamma}{p^r_M(\gamma + \beta)} & \text{for } \hat{p}_{N\min} \leq p^r_M \leq v \\
1 & \text{for } p^r_M > v 
\end{cases}
\]

(10)

with support \([\hat{p}_{N\min}, v]\) and has a mass point at \( v \) equal to

\[
m = \frac{\gamma}{\gamma + \beta},\]

(11)

where

\[
\hat{p}_{N\min} = \frac{v\gamma}{\gamma + \beta}.
\]

(12)
(ii) The discriminating-merged firm chooses a price randomly from the distribution

\[ G^r(\hat{p}_N) = \begin{cases} 
0 & \text{for } \hat{p}_N < \hat{p}_{N\min} \\
1 - \frac{(v - \hat{p}_N)\gamma}{\hat{p}_N\beta} & \text{for } \hat{p}_{N\min} \leq \hat{p}_N \leq v \\
1 & \text{for } \hat{p}_N \geq v
\end{cases} \]  

(13)

with support $[\hat{p}_{N\min}, v)$.

(iii) The profit for the discriminating firm from poached consumers equals

\[ \pi^r_M = \frac{v\gamma\beta}{\gamma + \beta}, \]  

and the profit for the non-discriminating firm equals $\pi_N = v\gamma$.

Proof. See the Appendix.

Therefore, total second-period expected profit for each of the insider discriminating firms, denoted $\pi^2_M$, equals

\[ \pi^2_M = \frac{1}{2} (\pi_Q^M + \pi^r_M) = \frac{1}{2} \left( 2v\gamma + \frac{v\gamma\beta}{\gamma + \beta} \right), \]  

(15)

while the second-period profit for the outsider non-discriminating firm is

\[ \pi^2_N = v\gamma. \]

As $\pi^2_M - \pi^2_N = (v\gamma\beta) / [2(\gamma + \beta)] > 0$ we have a measure of the benefit of embarking on a merger when price discrimination is permitted.

It should be remarked that in the equilibrium derived above, the outsider non-discriminating firm uses a "Hi-Lo" pricing strategy. To squeeze more surplus from its captive customers, it charges the highest price $v$, with probability $m$; and to avoid being poached and loosing the group of customers willing to switch it quotes occasionally a low price.

Corollary 3 From the equilibrium distribution functions defined by (10) and (13) it follows that:

(i) $\hat{G}(p_M^r) < G^r(\hat{p}_N)$, that is, $\hat{G}(p_M^r)$ first-order stochastically dominates $G^r(\hat{p}_N)$;
(ii) $E(\hat{p}_N) > E(p_M^r)$; and
(iii) the mass point $m$ is decreasing in $\beta$ and increasing in $\gamma$.

Proof. See the Appendix.

As expected, the outsider non-discriminating firm charges on average higher prices than the merged discriminating firm. After the merger, the discriminating firm has an advantage over its
rival because it is able to entice some of the rival’s customers to switch, without damaging the profit from its locked in segment. Conversely, the outsider firm cannot protect its captive market from price cuts. When it charges a low price, as a way to avoid poaching, it damages the profit from its captive segment. As the merged discriminating company has less to lose, it can be more aggressive and, therefore, it charges, on average, lower prices. Part (iii) states that the greater is the size of the outsider non-discriminating firm’s captive group, the higher is the probability of this firm charging the monopoly price \( v \). The reverse happens with respect to the size of the switchable segment.

Contrasting the results in Proposition 2 and 6, one immediately sees that a merger between firms with information to engage in behaviour-based pricing boosts the insiders’ second period profit and has no impact on the outsider’s second-period profit.

**One merging firm has information and the other does not**. In this scenario the merger of customer purchase databases will ‘obfuscate’ the merged entity which after the merger will not be able to distinguish all customer types (as was the case in the previous scenario). This will be the case when we have a merger between a firm that in period 1 sold only to its captive customers and a firm that in period 1 sold to both its captive customers and to shoppers. As before, with no loss of generality, suppose that in the first-period firms 1 and 2 sell only to their captive customers. Firm 3 is the lowest-price firm in period 1 and so serves its captive market as well as the shoppers. Suppose that we have a merger between firm 1 and 3. In this case, after first period decisions have been made, all of firm 1’s captive customers will have a record on firm 1’s purchase histories database. Similarly, all of firm 3’s captive customers as well as the shoppers will have a record on firm 3’s customer database. Now, in the second period, by joining firm 1’s and firm 3’s databases, the merged firm will only be able to recognize as being captive customers those that bought from firm 1 before. This means that the merged firm can (in period 2) charge two prices, one price tailored to those customers recognized as captive \( (p^c_M) \) and another one to all those customers which have a record on firm 3’s database \( (\tilde{p}_M) \). In contrast, the outsider firm (here firm 2) will be able to distinguish a previous captive customer from a price sensitive customer who bought from a rival before. The outsider firm will then charge, in period 2, two prices, one price targeted to its previous captive customers \( (p^c_O) \) and another one targeted to the rival’s previous customer \( (p^r_O) \).

**Lemma 4** The merged firm \( M \) will charge the customers recognized as captive customers the monopoly price, \( p^c_M = v \).

As the merged firm does not compete with the outsider firm with regards to part of its captive customers (those \( \gamma_1 \) consumers who bought from firm 1 in period 1) it has no incentive to charge them anything other than their reservation price. However, the merged firm has no way to distinguish between the group of customers that previously bought from firm 3 and the switchable customers. As in Proposition 5, there is no pure strategy equilibrium in prices for the group of consumers that bought from the lowest-price firm in period 1. There is, however, an asymmetric MSNE. Let \( G_O(\tilde{p}_M) \) denote the probability that the outsider firm’s price to the rival’s previous customers is no higher than \( \tilde{p}_M \) and \( G_M(p^c_O) \) denote the probability that the merged firm’s price is less than or equal to \( p^c_O \).
Proposition 7  Price competition over the group of consumers (shoppers) previously buying from the merged firm gives rise to an asymmetric MSNE in which:

(i) The merged firm chooses a price randomly from the distribution

\[
\tilde{G}_M(p^*_O) = \begin{cases} 
0 & \text{for } p^*_O < \tilde{p}_M \min \\
1 - \frac{\frac{\gamma}{p^*_O(\gamma + \beta)}}{p^*_M} & \text{for } \tilde{p}_M \min \leq p^*_O \leq v \\
1 & \text{for } p^*_O > v
\end{cases}
\]  

(16)

with support \([\tilde{p}_M \min, v]\) and has a mass point at \(v\) equal to

\[
\tilde{m} = \frac{\gamma}{\gamma + \beta},
\]  

(17)

where

\[
\tilde{p}_M \min = \frac{v\gamma}{\gamma + \beta}.
\]  

(18)

(ii) The outsider firm chooses a price randomly from the distribution

\[
G_O(\tilde{p}_M) = \begin{cases} 
0 & \text{for } \tilde{p}_M < \tilde{p}_M \min \\
1 - \frac{(v - \tilde{p}_M)\gamma}{\tilde{p}_M (\beta + \gamma)} & \text{for } \tilde{p}_M \min \leq \tilde{p}_M \leq v \\
1 & \text{for } \tilde{p}_M \geq v
\end{cases}
\]  

(19)

with support \([\tilde{p}_M \min, v]\).

(iii) The outsider firm’s profit from poached consumers equals

\[
\pi^*_O = \frac{v\gamma \beta}{\gamma + \beta}.
\]  

(20)

and the profit earned by the merged firm with its non-recognized customers \(\tilde{\pi}_M = v\gamma\).

Proof. See the Appendix. ■

Note that, as for the second-period, the profit earned by the merged firm is \(\pi^2_M = 2v\gamma\) whereas the outsider firm’s profits equals \(\pi^2_O = v\gamma + (v\gamma/\beta) / (\gamma + \beta)\).

Corollary 4  When a merger of customer databases discloses partial information about a customer type, the merger has no effect on the insiders’ second period profit but has a positive effect on the outsider firm’s profit.
Contrasting the results in Propositions 1 and 4, on the one hand, and those in Propositions 2, 6 and 7, on the other, one concludes that a merger will only occur in equilibrium when: (i) price discrimination is permitted; and (ii) the merger involves the two firms with the required information to distinguish a captive customer from a switchable one and price discriminate accordingly in the second period price game. This will be the case when we have a merger between firms selling exclusively to their captive customers in period 1. As before and with no loss of generality, suppose that firm 1 and 2 sold only to the segment of captive customers in period 1, meaning that they have the required information to recognize all types of customers in period 2. Formally, when price discrimination is permitted, firms 1 and 2 will both announce \( f_1; 2 \), whereas the lowest first-period price firm, firm 3, will announce a singleton coalition \( \{3\} \). Therefore, the resulting equilibrium coalition structure will be \( C = \{\{1, 2\}, 3\} \).

4.2.2 First-Period

Consider next the equilibrium first-period pricing. Firms make their pricing choices simultaneously and rationally anticipating how such decisions will affect both the merger game outcome and their profits in the subsequent period. Again, a similar reasoning as in Varian (1980) and Narasimhan (1988) shows that there is no subgame perfect Nash equilibrium in pure strategies. There is, however, a MSNE, the existence of which is proved by construction. We have already seen that the firm charging the lowest price in period 1 does not embark in a merger in equilibrium. Therefore, for a given price \( p_i \) chosen by firm \( i \) in period 1, firm \( i \) is the lowest-price firm in period 1 (or the non-discriminating outsider firm in period 2) with a probability equal to \( \prod_{j \neq i} \left[ 1 - F_j^1 (p_i) \right] \). On the other hand, firm \( i \) is one of the discriminating insider firms in period 2 with a probability equal to \( 1 - \prod_{j \neq i} \left[ 1 - F_j^1 (p_i) \right] \), where \( F_j^1 (p_i) \) denotes the probability that firm \( j \)'s price in period 1 is less than or equal to \( p_i \). Since we are looking for a symmetric MSNE, let \( F_j^1 (p) = F^1 (p) \) for all firms.

Overall expected profit for firm \( i \) when it charges first-period price \( p_i \), uses a discount factor equal to \( \delta \), and its competitors price according to \( F^1 (p_i) \), is equal to:

\[
E \Pi_i = p \gamma \left\{ 1 - \left[ 1 - F^1 (p) \right]^2 \right\} + p \left( \gamma + \beta \right) \left[ 1 - F^1 (p) \right]^2 \\
+ \delta \left[ \left[ 1 - F^1 (p) \right]^2 \pi_M + \left[ 1 - \left[ 1 - F^1 (p) \right]^2 \right] \frac{1}{2} \pi_N \right].
\]

Equivalently,

\[
E \Pi_i = p \gamma + \delta \left( \frac{v \gamma}{2 (\gamma + \beta)} \right) + \left[ 1 - F^1 (p) \right]^2 \left( \frac{p \beta - \frac{\delta v \gamma \beta}{2 (\gamma + \beta)}}{2 (\gamma + \beta)} \right).
\]  

\footnote{In all other cases, the merger does not enhance insiders’ (aggregate) profits and, hence, will not occur in equilibrium.}
Proposition 8 When price discrimination is permitted and a two-firm merger occurs, there is a symmetric subgame perfect MSNE in which:

(i) Each firm’s first-period price is randomly chosen from the distribution

\[
F^1(p) = \begin{cases} 
0 & \text{for } p \leq p_{\text{min}} \\
1 - \left( \frac{(v-p)\gamma}{p\beta - \delta} \left( \frac{v\gamma}{2(\gamma + \beta)} \right) \right)^{1/2} & \text{for } p_{\text{min}} \leq p \leq v \\
1 & \text{for } p \geq v 
\end{cases}
\]

with minimum equilibrium price equal to

\[
p_{\text{min}} = \frac{v\gamma}{\gamma + \beta} + \delta \frac{v\gamma\beta}{2(\gamma + \beta)^2};
\]

(ii) Each firm earns expected overall equilibrium profits equal to

\[
E(\Pi^*) = v\gamma + \delta \left( v\gamma + \frac{v\gamma\beta}{2(\gamma + \beta)} \right)
\]

Making use of Propositions 3 and 8, it follows that the merger possibility gives rise to a positive effect on individual firm’s expected overall profits:

\[
E(\Pi^*) - E(\Pi) = v\gamma + \delta \left( v\gamma + \frac{v\gamma\beta}{2(\gamma + \beta)} \right) - (1 + \delta)v\gamma = \frac{1}{2} \frac{v\gamma\beta}{\beta + \gamma}.
\]

As we will see in the next section, this result is due to the expected price effects of mergers when information-based price discrimination is possible.

5 Price effects of mergers when price discrimination is possible

In this section, we investigate the induced effects of mergers both on first period prices and on second period prices.

Second-period price effects When price discrimination is possible, the merged discriminating firm increases (or at least does not reduce) the price to its captive consumers who will pay the monopoly price \(v\). These consumers are expected to pay a higher price in period 2. From a comparison between \(F^1\) and \(G^r\), we observe that \(F^1 < G^r\). Therefore, the first-period price is stochastically larger than \(p^r\). It follows that, if poaching occurs, the group of shoppers will pay, on
average, a lower second-period price. Look next at the second period price charged to customers that are only aware about the outsider non-discriminating (low first-period price) firm. Here the conclusion is less clear-cut. We have seen that this firm will use a “Hi-Lo” pricing strategy in period 2. With probability equal to \( m \), its locked-in customers will pay the monopoly price \( v \) in period 2. Otherwise, because it is not possible to establish a general stochastic order between \( F^1 \) and \( G \), this set of consumers may end up paying a higher or lower second-period price.

**First-period price effects** The next table summarizes the equilibrium first-period price distribution in four possible scenarios which depend on whether price discrimination and/or mergers are possible.

<table>
<thead>
<tr>
<th>No Discrimination</th>
<th>Discrimination</th>
</tr>
</thead>
<tbody>
<tr>
<td>No Merger</td>
<td>Merger</td>
</tr>
<tr>
<td>( F = 1 - \left( \frac{(v-p)\gamma}{p^\beta} \right)^{\frac{1}{2}} )</td>
<td>( F = 1 - \left( \frac{(v-p)\gamma}{p^\beta} \right)^{\frac{1}{2}} )</td>
</tr>
<tr>
<td>No Merger</td>
<td>Merger</td>
</tr>
<tr>
<td>( F = 1 - \left( \frac{(v-p)\gamma}{p^\beta} \right)^{\frac{1}{2}} )</td>
<td>( F^1 = 1 - \left( \frac{(v-p)\gamma}{p^\beta - \delta} \right)^{\frac{1}{2}} )</td>
</tr>
</tbody>
</table>

**Table 1: First-Period Equilibrium Price Distributions**

From the equilibrium distribution functions it immediately follows that the effect of a merger on first-period prices depends on whether price discrimination is permitted or not. Similarly, the effect of price discrimination on first-period prices depends on whether a merger to duopoly can or cannot occur.

**Corollary 5** *From the comparison between \( F \) and \( F^1 \), it follows that \( F^1 < F \), that is \( F^1 \) first order stochastically dominates \( F \) as long as \( \delta > 0 \). Therefore, \( E(p^1) > E(p) \).*

We have seen that when price discrimination and mergers are permitted, there is a benefit of embarking on a merger when both merging firms are informed. This then suggests that firms have a strategic incentive to quote high first-period prices as a way to secure being one of the insider (discriminating) firms in the second period. This acts to soften first-period price competition and so, under discrimination, firms are expected to charge higher first-period prices.\(^{24}\) Note also that the support of equilibrium prices is \( \left[ \frac{v^\gamma}{\gamma + \beta}, v \right) \) both when there is no price discrimination and when there is price discrimination but mergers cannot occur. In contrast, the support of equilibrium prices is \( \left[ \frac{v^\gamma}{\gamma + \beta} + \delta \frac{v\gamma \beta}{2(\gamma + \beta)}, v \right) \) when both mergers and price discrimination are allowed. It is straightforward to see that, as \( \delta > 0 \), there is less price dispersion in the latter case.

In sum, in industries where information-based price discrimination is likely to occur a merger to duopoly will lead firms to charge higher first period prices and also higher second period prices. This competition softening effect boosts the overall equilibrium expected profit of all firms in the status quo industry structure (see eq. (24)).

\(^{24}\) A similar result is obtained in Chen and Zhang (2009) and also in Esteves (2009) with regards to the effects of price discrimination in a duopoly market.
6 Welfare Issues

This section evaluates the welfare effects of mergers when price discrimination is permitted. Although prices play no welfare role here—due to the unit demand assumption, no dropping out of consumers and no loyalty (or transport) costs—, price discrimination being permitted affects the firms’ merger decisions and so their profits and consumer welfare. Since production costs are assumed to be zero, total welfare \( W \) is equal to the value of the good for all buyers that enter the market in both periods, that is \( W = 2v \). Due to the previous assumptions, a merger will have no effect on overall welfare. Nevertheless, it is important to investigate separately the effects of mergers on industry profit and consumer welfare.

**Proposition 9** When price discrimination and mergers are permitted, industry profit increases at the expense of consumer surplus.

Without loss of generality, suppose that \( \delta = 1 \). To evaluate the profit and consumer surplus effects of mergers, we first analyze the case where price discrimination is permitted and we move from the no-merger to the merger scenario. As welfare is constant, the effect of a merger is to give rise to a transfer of income and wealth from individual consumers to the firms. When firms are not allowed to merge, then, from Proposition 3, it follows that industry profit is equal to \( \pi_{ind}^N = 6v\gamma \). This being the case, consumer surplus equals

\[
CS^N = 2v - 6v\gamma = 2v (1 - 3\gamma) = 2v\beta. \tag{25}
\]

When mergers are instead permitted, then, from Proposition 8, we obtain that there is a positive net effect on industry profit equals

\[
\pi_{ind}^M - \pi_{ind}^N = \frac{3v\gamma\beta}{2(\gamma + \beta)}. \tag{26}
\]

This gain is exactly compensated by a loss in terms of consumer surplus:

\[
CS^M - CS^N = -\frac{3v\gamma\beta}{2(\gamma + \beta)}. \tag{27}
\]

Our results have important policy implications. When mergers are not permitted, price discrimination has no overall effect on consumer welfare. Although price discrimination is good for shoppers who pay the marginal cost (with 3 firms), the gain due to price discrimination for this group of consumers is exactly compensated by the loss of captive consumers who end up paying the monopoly price.

This finding does not hold, however, when mergers are permitted. As stated in Proposition 9, when both price discrimination and mergers are allowed, a merger leads to an increase in industry profit at the expense of consumer surplus, leaving total welfare unaffected. This suggests
that competition authorities should scrutinize with greater zeal the mergers in industries wherein firms are expected to engage in BBPD. This result is therefore in stark contrast with the general presumption of Chen (2005), according to whom “price discrimination by purchase history ... is by and large unlikely to raise significant antitrust concerns. In fact, as the economics literature suggests, such pricing practices in oligopoly markets often intensify competition and potentially benefit consumers.” (p. 123).

Our model shows that, when firms are allowed to price discriminate, then the unique equilibrium merger reduces competition in such a way as to transfer wealth from customers to the merged firm (and its competitors). If total welfare is the criterion adopted by the competition authorities to appraise a proposed merger, the merger is welfare-neutral. However, a merger between two firms with information to engage in BBPD gives rise to price increases to consumers in both periods and, therefore, to consumer welfare losses. So if consumer surplus is the competition authority welfare standard, as it is the case in most antitrust jurisdictions, then this merger should give rise to serious antitrust concerns and should, therefore, be blocked.

7 Conclusion

The economics literature on oligopoly price discrimination by purchase history is relatively new and has focused mostly on markets with two symmetric firms, where the possibility of horizontal mergers is not considered. In these situations, dynamic price discrimination by competing firms often results in intensified competition; and such pricing practices are typically believed not to raise antitrust concerns.

This article has taken a first step in investigating the impact of Behaviour-Based Price Discrimination (BBPD) in markets where horizontal mergers may occur. By so doing, some important antitrust policy implications are obtained. First, it is shown that due to the ability of firms to price discriminate, the so called ‘free-riding problem’ identified by the previous horizontal mergers literature is eliminated. Second, and perhaps most importantly, by considering the possibility that firms embark on horizontal mergers, it is shown that such price discrimination practices (also in the form of customer poaching) can have a negative effect on consumers’ welfare. This then suggests that competition authorities should be particularly vigilant with regards to mergers in industries wherein firms are expected to engage in BBPD.

In concluding, it should be pointed out that an important limitation to the previous analysis is the fact that consumers are assumed to be naive and, therefore, do not foresee that the firm may react strategically to their initial choices. As Amstrong (2006) highlights, this assumption might be relevant for a new market, where consumers’ have not yet learned the firms’ pricing strategies. Clearly, however, this naivety assumption restricts the applicability of the proposed model, since consumers who have been active in the market for some time may anticipate the effect their current actions will have on their subsequent deals. So, it seems important to extend the analysis in order to consider the case of sophisticated consumers. This will be done in our future research. Hopefully, the above model can be seen as a stepping stone in the direction of a more complete analysis.
A Proofs

Proof of Proposition 4: Suppose that an asymmetric mixed strategy involves firms charging a price no higher than \( p \) with probability \( H_j(p), \ j = M, O \). If firm \( M \) chooses price \( p \) while the outsider uses mixed strategy, its expected profit is:

\[
E\pi_M = 2p\gamma + p\beta \left[1 - H_O(p)\right]. \tag{28}
\]

In equilibrium firm \( M \) must be indifferent between quoting any price that belongs to the equilibrium support, where \( p_M \in [p_{M \min}, v] \). Thus

\[
2p\gamma + p\beta \left[1 - H_O(p)\right] = 2v\gamma
\]

from which we obtain:

\[
H_O(p) = 1 - \frac{2\gamma(v - p)}{p\beta}
\]

\[
1 - \frac{2\gamma(v - p)}{p\beta} = 0
\]

where \( p_{M \min} = \frac{2v\gamma}{\beta + 2\gamma} \). As this firm would never want to price below \( p_{M \min} \), by quoting a price \( p_O \) arbitrarily close to \( p_{M \min} \), firm \( O \) poaches all the shoppers guaranteeing itself a profit of \( p_{M \min}(\beta + \gamma) = \frac{2v\gamma}{\beta + 2\gamma}(\beta + \gamma) \). (Note that \( \frac{2v\gamma}{\beta + 2\gamma}(\beta + \gamma) > v\gamma \) as long as \( \beta > 0 \)). Thus, any price \( p_{O \min} < p_{M \min} \) is a dominated strategy for firm \( O \). Thus, if firm \( O \) chooses price \( p \) while the other firm uses mixed strategy, its expected profit is:

\[
E\pi_O = p\gamma + p\beta \left[1 - H_M(p)\right]. \tag{29}
\]

In a MSNE we must observe that

\[
p\gamma + p\beta \left[1 - H_M(p)\right] = (\gamma + \beta) \frac{2v\gamma}{\beta + 2\gamma}
\]

from which we obtain \( H_M(p_{M \min}) = 0 \) and \( H_M(v) = 1 - \frac{\gamma}{\beta + 2\gamma} \). Thus firm \( H_M(p) \) has a mass point at \( v \) equal to \( \frac{\gamma}{\beta + 2\gamma} \).

Proof of Proposition 5: Suppose \( (p^r, \widehat{p}_N) \) is an equilibrium in pure strategies. Then, by definition, there is no such \( p^r \), such that \( \pi_M(p^r_M, \widehat{p}^r_N) > \pi_M(p_M, \widehat{p}_N) \). The proof proceeds by contradiction.
(i) If $p_{M}^{*} = \hat{p}_{N}$, then

$$\pi_{i}^{*} = \frac{1}{2} \beta p_{M}^{*}.$$  \hspace{1cm} (30)

If firm $M$ deviates and quotes $p_{M}^{*} = \hat{p}_{M}^{*} - \varepsilon$, with $\varepsilon > 0$, its profit from deviation is $\pi_{M}^{*} = \beta (p_{M}^{*} - \varepsilon)$. It is then trivial to see that there exists such an $\varepsilon$ that makes the deviation profitable. A contradiction. $Q.E.D.$

(ii) Let $p_{M}^{*} < \hat{p}_{N}$ then

$$\pi_{i}^{*} = \beta p_{i}^{*}.$$  \hspace{1cm} (31)

Let $p_{M}^{*} = \hat{p}_{M}^{*} + \varepsilon < \hat{p}_{N}$, then, firm $i$’s profit from deviation is $\pi_{i}^{*} = \beta (p_{i}^{*} + \varepsilon)$, from which it is straightforward to see that the deviation is profitable. A contradiction. $Q.E.D.$

**Proof of Proposition 6:** The existence of such an equilibrium is proved by construction. It is a dominated strategy for each firm to set a price above $v$. Additionally, the non-discriminating firm can guarantee itself a profit of $v\gamma$, charging $v$ to its captive customers. It thus follows that at price $\hat{p}_{N}$ the best it can do is to attract all shoppers as well as its captive customers. This means that a necessary condition for it to be willing to charge $\hat{p}_{N}$ is:

$$\hat{p}_{N} (\gamma + \beta) \geq v \gamma.$$  

In other words, any $\hat{p}_{N} < \hat{p}_{N \min} = \frac{v \gamma}{\gamma + \beta}$ is a dominated strategy for the non-discriminating firm. As this firm would never want to price below $\hat{p}_{N \min}$, by quoting a price $p_{M}^{*}$ arbitrarily close to $\hat{p}_{N \min}$, the discriminating firm poaches all the selective customers that bought previously from the rival, guaranteeing itself a profit of $\hat{p}_{N \min} \beta = \frac{v \gamma \beta}{\gamma + \beta}$. Thus, any price $p_{M}^{*} < \hat{p}_{N \min}$ is a dominated strategy for the discriminating firm.

Next, we prove that neither firm has a mass point $p^{*}$, such that $\hat{p}_{N \min} < p^{*} < v$. By way of contradiction, assume that $p^{*}$ is chosen with positive probability by firm the discriminating firm. Then by choosing $\hat{p}_{N} = p^{*} - \varepsilon$, where $\varepsilon$ is arbitrarily small, the non-discriminating firm becomes the low priced firm and can increase its profits. There is a profitable deviation. A contradiction. Assume now that that $p^{*}$ is chosen with positive probability by the non-discriminating firm. Then by choosing $p_{M}^{*} = p^{*} - \varepsilon$, where $\varepsilon$ is arbitrarily small, the discriminating firm has a profitable deviation. A contradiction. By similar arguments it is also straightforward to show that neither firm has a mass point at $\hat{p}_{N \min}$. It remains to prove that only the non-discriminating firm has a mass point at the highest price $v$. If the non-discriminating firm has a mass point at $v$, the discriminating firm is always better off not charging that price but coming arbitrarily close to it. Following Narasimhan (1988) it is also straightforward to prove that both distribution functions are strictly increasing and continuous over the interval with lower bound $\hat{p}_{N \min}$ and upper bound $v$. In equilibrium, for the non-discriminating firm, the following condition must be satisfied:

$$\hat{p}_{N^{\gamma}} + \hat{p}_{N \beta} [1 - G^{\gamma} (\hat{p}_{N})] = v \gamma$$

25
It follows that

\[ G^r(\hat{p}_N) = 1 - \frac{(v - \hat{p}_N) \gamma}{\hat{p}_N \beta}, \]  

(32)

with \( G^r(\hat{p}_{N \text{min}}) = 0 \) and \( G^r(v) = 1 \). This proves part (ii).

Similarly, in equilibrium, the discriminating firm must be indifferent between prices that belong to the half open interval \([\hat{p}_{N \text{min}}, v)\), i.e.:

\[ p_M^r \beta \left[ 1 - \hat{G}_N(p^r) \right] = \hat{p}_{N \text{min}} \beta \]

from which it follows that:

\[ \hat{G}_N(p^r_M) = 1 - \frac{\hat{p}_{N \text{min}}}{p^r_M} = 1 - \frac{v \gamma}{p^r_M (\gamma + \beta)}, \]

(33)

with \( \hat{G}_N(p^r_M = \hat{p}_{N \text{min}}) = 0 \) and \( \hat{G}_N(v) = 1 - \frac{\gamma}{\gamma + \beta} \) which is smaller than 1 as long as \( \beta > 0 \) which by assumption is always true. This implies that the non-discriminating firm has a mass point at \( v \). This completes the proof. \( Q.E.D. \)

**Proof of Proposition 7:** The existence of such an equilibrium is proved by construction. It is a dominated strategy for each firm to set a price above \( v \). Additionally, the merged firm can guarantee itself a profit of \( v \gamma \), charging \( v \) to its unrecognized captive customers. It thus follows that at price \( \bar{p}_M \) the best it can do is to attract all shoppers as well as its unrecognized captive customers. This means that a necessary condition for it to be willing to charge \( \bar{p}_M \) is:

\[ \bar{p}_M (\gamma + \beta) \geq v \gamma. \]

In other words, any \( \bar{p}_M < \bar{p}_{M \text{min}} = \frac{v \gamma}{\gamma + \beta} \) is a dominated strategy for the merged firm. As this firm would never want to price below \( \bar{p}_{M \text{min}} \), by quoting a price \( p^r_O \) arbitrarily close to \( \bar{p}_{M \text{min}} \), the outsider firm poaches all the switchable customers that bought previously from the rival, guaranteeing itself a profit of \( \bar{p}_{M \text{min}} \beta = \frac{v \gamma \beta}{\gamma + \beta} \). Thus, any price \( p^r_O < \bar{p}_{M \text{min}} \) is a dominated strategy for the outsider firm. Following a similar proof as in the proof of proposition 6, it is straightforward to prove that only the merged firm can have a mass point at \( v \). In equilibrium, for the merged firm, the following condition must be satisfied:

\[ \bar{p}_M \gamma + \bar{p}_M \beta \left[ 1 - G^r_O(\bar{p}_M) \right] = v \gamma \]

from which we obtain:

\[ G^r_O(\bar{p}_M) = 1 - \frac{(v - \bar{p}_M) \gamma}{\bar{p}_M \beta}, \]

(34)

It thus follows that \( G^r_O(v) = 1 \) and from \( G^r_O(\bar{p}_{M \text{min}}) = 0 \) we obtain \( \bar{p}_{M \text{min}} = \frac{v \gamma}{\gamma + \beta} \).
Similarly, in equilibrium, the outsider firm must be indifferent between prices that belong to the interval \([\bar{p}_{M_{\min}}, v]\), i.e.:

\[
p_{O}^{\beta} \left[ 1 - \tilde{G}_{M}(p_{O}^{\beta}) \right] = \bar{p}_{M_{\min}}^{\beta}
\]

It follows that:

\[
\tilde{G}_{M}(p_{O}^{\beta}) = 1 - \frac{\bar{p}_{M_{\min}}}{p_{O}^{\beta}} = 1 - \frac{v_{\gamma}}{p_{O}^{\beta} (\gamma + \beta)}
\]  

(35)

with \(\tilde{G}_{M}(p_{O}^{\beta} = \bar{p}_{M_{\min}}) = 0\) and \(\tilde{G}_{M}(v) = 1 - \frac{\gamma}{(\gamma + \beta)}\), smaller than 1 as long as \(\beta > 0\) which by assumption is always true. This implies that the merged firm has a mass point at \(v\) equal to \(\bar{m} = \frac{\gamma}{\gamma + \beta}\). This completes the proof. \textit{Q.E.D.}

**Proof of Corollary 3:** To prove part (i) note that \(G'(\bar{p}_{N}) - \tilde{G}_{N}(p_{M}^{\beta})\) can be written as \(\left[\frac{v - p}{\beta} - \frac{v}{\gamma + \beta}\right] \frac{\gamma}{p}\). Since \(\frac{\gamma}{p} > 0\) and \(\beta (\gamma + \beta) > 0\), then \(G'(\bar{p}_{N}) - \tilde{G}_{N}(p_{M}^{\beta}) > 0\) as long as \((v - p) \gamma > 0\), which is always true. When (i) holds, result (ii) follows. \textit{Q.E.D.}

**Proof of Proposition 8:** The overall expected profit for firm \(i\), when it charges first-period price \(p_{i}\), uses a discount factor equal to \(\delta\), and their competitors charge a first-period price equal to \(p_{j}\) according to \(F_{1}^{j}(p_{i})\), is equal to:

\[
E\Pi_{i} = p_{i} \left\{ 1 - \left[ 1 - F_{1}^{1}(p) \right]^{2} \right\} + p_{i} (\gamma + \beta) \left[ 1 - F_{1}^{1}(p) \right]^{2} + \delta \left[ 1 - F_{1}^{1}(p) \right]^{2} \pi_{N} + \left[ 1 - \left[ 1 - F_{1}^{1}(p) \right]^{2} \right] \frac{1}{2} \pi_{M}.
\]

\[
E\Pi_{i} = p_{\gamma} + \delta \left( v_{\gamma} + \frac{v_{\gamma}\beta}{2 (\gamma + \beta)} \right) + \left[ 1 - F_{1}^{1}(p) \right]^{2} \left( p_{\beta} - \frac{\delta v_{\gamma}\beta}{2 (\gamma + \beta)} \right)
\]

In MSNE the firm must be indifferent between quoting the monopoly price \(v\) or any price in the equilibrium support.

\[
E\Pi_{i} = p_{\gamma} + \delta \left( v_{\gamma} + \frac{v_{\gamma}\beta}{2 (\gamma + \beta)} \right) + \left[ 1 - F_{1}^{1}(p) \right]^{2} \left( p_{\beta} - \frac{\delta v_{\gamma}\beta}{2 (\gamma + \beta)} \right) = v_{\gamma} + \delta \left( v_{\gamma} + \frac{v_{\gamma}\beta}{2 (\gamma + \beta)} \right)
\]

Solving for \(F_{1}^{1}(p)\)

\[
F_{1}^{1}(p) = 1 - \left( \frac{(v - p) \gamma}{p_{\beta} - \delta \left( \frac{v_{\gamma}\beta}{2 (\gamma + \beta)} \right)} \right)^{\frac{1}{2}}
\]

Given that \(F_{1}^{1}(p_{\min}) = 0\) we find that \(p_{\min} = \frac{v_{\gamma}}{\gamma + \beta} + \delta \frac{v_{\gamma}\beta}{2 (\gamma + \beta)}. \textit{Q.E.D.}\)
References


